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Fool's Logic: The Shared Common Knowledge Multi-agent System's Model

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Abstract

In this paper, we give a formalization for multi-agent reasoning systems based on shared common knowledge views. We assume that the multi-agent systems have the following characters: There is a fool agent which represents the common knowledge base, and other agents are normal agents. Only fool agent's knowledge is common knowledge. All the axioms in traditional logic are regarded as common knowledge and hence can be used by any agent. Classical necessitation inference rules are not included in our logic systems, $\{W, W4, W5\}$. All of them share the same properties. For example: They are inductive, monotonic, complete and self-closed under inference. We compare their relationship with the traditional logical system $\{KD4, S4, S5\}$. Main conclusions are: Traditional modal logic inference can be realized by a fool agent. And our new logic can mimic the traditional logic inference correctly.

1 Introduction

Knowledge representation and reasoning in Multi-Agent Reasoning System (MARS) has been attracting much more attention in recent years [6] [5] [13] [23]. Generally, the logic formalizations of MARS are based on classical modal logics, such as S5, S4, and have formed a class of multi-agent knowledge and belief systems. There are different opinions about the concepts of knowledge and belief. The main difference are:

1. Should any agent's knowledge have to be true?

That is, should the axiom $Kip \rightarrow p$ be included in the logic formalization? Generally, it is agreed that if a logic system contains this axiom, then it is called a knowledge system, otherwise it is called a belief system. So the main difference between knowledge and belief is that the knowledge should be objectively true.

2. Should any agent have negative and positive introspective ability?

This question can be studied in two ways.

2.1 One approach is to discuss whether the positive and negative axioms should be included in the logic systems.

Where $Kip \rightarrow KiKip$ is called a positive axiom and $\neg Kip \rightarrow Ki\neg Kip$ is called a negative axiom.

2.2 Another approach is generally discussed in non-monotonic reasoning fields, where the negation as failure concept was introduced [17] [16].

For example, as in [17], if p is not included in an agent's conclusion set (AEL extension), then $\neg Kip$ is included in the agent's conclusion set. For pure logic study reasons, we only consider the first direction in this paper.

The knowledge axiom is included in W5 and W4. The negative introspective axiom is only included in W5. W, which was first proposed in [23] and improved in [27], contains only the positive axiom, knowledge distributed axiom and D axiom.

3. Should the real world knowledge be known by any agent?

Does this mean that we should accept the necessitation rule, $p \Rightarrow Kip$, in our new logic system?

We do not accept this necessitation inference rule in our logic system class. The reason is very simple. For example, if agent i knows p, then agent j should not logically know that agent i knows p.

One of the typical properties of MARS is its societies, in which common knowledge is an important topic in recent years [1] [5] [6] [7]. According to [5], [6], say p is a common knowledge in agent group Ag iff

1. Every agent in Ag knows p.

2. For every agent i in Ag, i knows that 'p is a common knowledge'.

Formalization about common knowledge in modal logic S5 can be found in [6] [5], in which a common knowledge modal operator C_{Ag} is introduced. Suppose $E_{Ag}p$ denotes $\bigwedge_{i \in Ag} K_i p$, then the additional axioms and inference rule to S5 are:

C1: $E_{Ag}p \equiv \bigwedge_{i \in Ag} K_i p$.

C2. $C_{Ag}(p \to q) \to (C_{Ag}p \to C_{Ag}q)$.

C3. $C_{Ag}p \equiv E_{Ag}(p \wedge C_{Ag}p)$

RC1. $p \to E_{Ag}p \Rightarrow p \to C_{Ag}p$.

C3 is called C_{Ag}'s fixed point axiom, and RC1 is the common knowledge inductive inference rule.It is proved in [6] that the common knowledge S5 system (multi-agent logic S5 system plus above axioms and inference rule) is complete. The problem with this research is that they give us the definition for common knowledge, but does not tell us how to use common knowledge. In fact, one conclusions of this research is that [?] since there is no safe communication or there are no perfect clocks, i.e. no truly simultaneous access to communication channels, there is impossible to attain common knowledge.

In fact, in real life, common knowledge is not so strict. For example, in the broadcasting system, the knowledge being broadcast can be regarded as common knowledge. The knowledge in a shared common knowledge base can also be regarded as common knowledge, since every agent can reach it, and every agent knows that the others can reach it. So it is necessary to set up a new logic system to describe such kinds of common knowledge. Such kind of distributed knowledge can be described as a set of agents sharing a common knowledge base. Every agent can have its own knowledge, and the knowledge in a common knowledge base can be reached by any agent, and every one knows it. Such a MARS model is called the shared common knowledge MARS model.

What is the main difference between our common knowledge and the traditional concept? I think there are two fundamental differences.

One is our definition that common knowledge is for use. So, for example, we assume that tautology should be common knowledge, knowledge distributed axiom, positive introspective axiom should be common knowledge.

Another difference is that traditional common knowledge emphasizes that if p is a common knowledge, then every agent knows it, and every agent knows 'it is a common knowledge'. That is p is a common knowledge iff For every agent i, 1. i knows p, and 2. i knows 'p is common knowledge'. This is the so-called common knowledge's fixed point axiom. Our understanding about common knowledge is a little different. We say if p is a common knowledge, then 1. every agent knows it, and 2. 'every agent knows it' is also a common knowledge. It is the second difference which makes our logic system a system for using common knowledge.

In this paper, a logic class $\{W, W4, W5\}$ has been established to formalize the shared common knowledge MARS model. The common knowledge base is characterized by a fool agent which is first introduced in [15], and also appeared in [10], [23]. Typical properties of this logic class are:

1. Every axiom in this class is common knowledge (in form of K0..).
2. Necessitation rule is not included in this class.
3. Safe rule $K0p \Rightarrow p$ is introduced to show that every common knowledge p is true in the real world.

Logics introduced in this paper can also be regarded as a fool's logic in a multi-agent reasoning environment. For example, the knowledge distributed axiom $Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq)$ is not only a conclusion of our logic system, but also a common knowledge in our system. This means that every agent can be aware that it and the others know this knowledge.

This paper is organized as follows. In section 2, we introduce the propositional shared common knowledge MAR logic class $\{W, W4, W5\}$. We discuss the common properties of this class, and get some important properties. Suppose $X \in \{W, W4, W5\}$, then the main properties are: 1. Every X has the deductive property. Suppose T is a theory, p, q are two formulas, then $T \cup \{p\} \vdash_X q$ iff $T \vdash (p \rightarrow q)$. 2. For every formula p, $\vdash_X p$ iff $\vdash_X K0p$. 3. Suppose T is a theory and $T = Cons_X(T)$, then for every agent $i \in Ag$, $T/Ki = Cons_X(T/Ki)$. 4. X is consistent. In section 3, we introduce the model semantics of X. We prove the soundness of X under these semantics. And in section 4, we prove X's completeness using canonical X-Kripke structure. In section 5, we introduce the traditional knowledge and belief systems, S5, S4 and KD4. Some of the properties are listed without proof. More details about these properties proof can be found in [2]. In section 6, we concentrate our attention on the relationship between class $\{W, W4, W5\}$ and class $\{KD4, S4, S5\}$. Main results are: Suppose f is a map from $\{W, W4, W5\}$ to $\{KD4, S4, S5\}$, such that $f(W) = KD4$, $f(W4) = S4$, $f(W5) = S5$, S is a theory and q is a formula which contains no modal operator, then 1. If $S \vdash_X q$ then $S \vdash_{f(X)} q$. 2. If $S \vdash_{f(X)} q$ then $K0S \vdash_X K0q$. The first conclusion says that when we do not consider common knowledge, our logic system can be expressed in traditional logic; The second conclusion says that traditional modal inference can be correctly performed by the fool agent in our logic systems. So the logic class proposed in this paper is much stronger then traditional knowledge and belief MARS logic.

2 Class of the Multi-agent System based on Fool Reasoner

Suppose At is a set of primitive statements. $Ag = \{0, 1, ..., n\}$ is the set of agents, in which 0 is called the fool agent, the rest is called the normal agent or agent if it is not confused. Informally, 0's knowledge is common knowledge, which is known by all agents.

First, we define the syntax of the well-founded formulas based on At and Ag.

Definition 2.1 A well-founded formula based on At and Ag can be inductively defined as follows:

1. If $p \in At$, then p is a well-founded formula.

2. If p, q are well-founded formulas, $i \in Ag$, then Kip, $(\neg p), (p \rightarrow q)$ are also well-founded formulas.

3. All well-founded formulas are defined by the finite compositions of steps 1 and 2. □

We denote the set of all the well-founded formulas based on At and Ag, by L.

We use special symbols to abbreviate some formulas. We write $(p \vee q)$ for $(\neg p \rightarrow q)$, $p \wedge q$ for $\neg(p \rightarrow \neg q)$, $p \equiv p$ for $(p \rightarrow q) \wedge (q \rightarrow p)$. Assume formula P to be a basic formula if P contains no modal operator.

The axioms and inference rules of W5 are defined as follows.

Definition 2.2 W5's axioms:

A1. $K0p$, if p is any tautology.

A2. $K0(K0p \rightarrow K0Kip)$.

A3. $K0(Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$.

A4. $K0(K0p \rightarrow p)$.

A5. $K0(Kip \rightarrow KiKip)$.

A6. $K0(Ki\neg p \rightarrow \neg Kip)$.

A7. $K0(Kip \rightarrow p)$.

A8. $K0(\neg Kip \rightarrow Ki\neg Kip)$

W5's inference rules are:

Modus Ponens: $p, p \rightarrow q \Longrightarrow q$

Safeness rule: $K0p \Longrightarrow p$ □

Definition 2.3 W and W4 Logical Systems.

W4 is the logical system of S5 deleting the axiom scheme 8.

W is the logical system of S4 deleting the axiom scheme 7. □

Notice that, the safeness rule can be excluded from W if we add a new axiom $K0p \rightarrow p$ to W.

Suppose $X \in \{W5, W4, W\}$.Wwe will discuss the common properties of logical system X in the following.

Definition 2.4 Extension $Cons_X$.

Suppose theory $T \subseteq L$. We define $Cons_X(T)$, the extension of T under logical system X, to be the smallest subset of L that satisfies the following conditions:

1. $T \cup$ Axioms(X) $\subseteq Cons_X(T)$

2. If $p \in Cons_X(T), p \rightarrow q \in Cons_X(T)$ then $q \in Cons_X(T)$

3. If $K0p \in Cons_X(T)$ then $p \in Cons_X(T)$

□

Obviously, the concept of extension is well-defined and unique for every theory.

Now, it is not difficult to prove the following theorem.

Theorem 2.1 *Constructive property of $Cons_X(T)$.*

Suppose T is a theory. We can inductively construct the following sets:

$Cons^0_X(T) = Axioms(X) \cup T$, *and for all* $i \geq 0$:

$Cons^{i+1}_X(T) = Cons^i_X(T) \cup \{q|$ *there are formula p such that* $\{p, p \to q\} \subseteq Cons^i_X(T)$, *or* $K0q \in Cons^i_X(T)\}$

Then $Cons_X(T) = \sum_{i=0}^{i=\infty} Cons^i_X(T)$ □

Suppose T is a theory. As in [2], we can define the prove relationship between T and well-formed formula p. We denote it by $T \vdash_X p$, where p is called the consequence of T. Obviously, the consequence set of T is $Cons_X(T)$. That is, $Cons_X(T) = \{p|T \vdash_X p\}$.

Definition 2.5 [Contradiction]

Say theory T is contradiction in X, if there is a formula p such that $T \vdash_X \neg(p \to p)$ □

Obviously, from the proof definition, we can get:

Theorem 2.2 *Compactness Theorem*

$p \in Cons_X(T)$ *iff there is a finite subset T' of T, such that* $p \in Cons_X(T)$ *Or equally* $Cons_X(T) = \bigcup\{Cons_X(T')|T' \subseteq T$ *and* T' *is finite* $\}$ □

Suppose T, T' are two sets of formulas. We write $T \vdash_X T'$ as the abbreviation: for every $p \in T', T \vdash_X p$. From the compactness theorem, we can easily get:

Corollary 2.3 Monotonicity of W.

Suppose $T1, T2, T3$ are sets of formulas, if $T1 \vdash_X T2, T2 \vdash_X T3$ then $T1 \vdash_X T3$. □

Theorem 2.4 *Deduction theorem*

Suppose T is a theory, p, q are two formulas, then $T \bigcup \{p\} \vdash_X q$ *iff* $T \vdash_X p \to q$. □

Proof:

If $T \vdash_X p \rightarrow q$ then it is obvious that $T \bigcup \{p\} \vdash_X q$. Suppose r1,...,rk, $(p \rightarrow q)$ is a proof sequence of $p \rightarrow q$ in theory T under logic X. Then r1, ..., rk, $(p \rightarrow q)$, p, q is a proof sequence of q in theory $T \cup \{p\}$ under logic X.

Suppose $T \bigcup \{p\} \vdash_X q$, now we proof $T \vdash_X p \rightarrow q$ according q's proof length.

If q's proof length is 1. then there are two cases to get q.

1. q is p. Then we can easy to prove $T \vdash_X p \rightarrow q$, since $(p \rightarrow p)$ is a tautology.

2. $q \in T$ or q is an axiom of X. Then q, $K0(q \rightarrow (p \rightarrow q)$, $(q \rightarrow (p \rightarrow q)$, $p \rightarrow q$ is a proof sequence. So $T \vdash_X p \rightarrow q$.

Inductively, suppose that the above statement holds when the proof length is not greater than t.

Let q's proof length be $t+1$.

There are three cases to get q.

1. q is an axiom or $q = p$ or $q \in T$. Then from above discussion, we can see that $T \vdash_X p \rightarrow q$.

2. q is get from $(p1 \rightarrow q)$ and p1. both of their proof lengths are not greater then t. According to the deductive assumption we have:

$T \vdash_X (p \rightarrow (p1 \rightarrow q))$, $T \vdash_X p \rightarrow p1$

Since $(p \rightarrow (p1 \rightarrow q)), (p \rightarrow p1) \vdash_X (p \rightarrow q)$, we get

$T \vdash_X p \rightarrow q$.

3. q is get from $K0q$ whose proof length is not greater then t. We have

$T \vdash_X p \rightarrow K0q$. Since $\vdash_X K0q \rightarrow q$, and $(p \rightarrow K0q)$, $(K0q \rightarrow q) \vdash_X p \rightarrow q$, we get

$T \vdash_X p \rightarrow q$

This concludes the above theorem.

Lemma 2.1 For every formula p,q, agent i, $Ki(p \rightarrow q) \vdash (Kip \rightarrow Kiq)$.

Proof:

The proof sequence is: $Ki(p \rightarrow q)$, $K0(Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$, $(Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$, $(Kip \rightarrow Kiq))$. □

Lemma 2.2 For every formula p, $\vdash_X p$ iff $\vdash_X K0p$.

Proof:

Obviously, if $\vdash_X K0p$ then $\vdash_X p$.

Suppose $\vdash_X p$, we should prove $\vdash_X K0p$. We prove it by p's proof length.

When p's proof length is 1. Then p must be X's axiom. So p is in the form of $K0p1$. Then $K0p$'s proof is: p, $K0(K0p1 \rightarrow K0K0p1)$, $(K0p1 \rightarrow K0K0p1)$, K0p. So $\vdash_X K0p$.

Suppose the above statement is true for all formulas whose proof length is not greater then t.

Let p's proof length be (t+1). There are three cases to get p.

1. p is X's axiom. Then we have already proved that $\vdash_X K0p$.

2. p is obtained from $(p1 \rightarrow p)$ and p1 whose proof length are not greater then t. According to the assumption, we have

$\vdash_X K0(p1 \rightarrow p)$ and $\vdash_X K0p1$.

From Lemma 2.1 we get: $\vdash_X K0p$.

3. p is obtained from $K0p$ whose length is not greater then t. Then it is obvious that $\vdash_X K0p$.

Thus ends our deductive proof. □

Corollary 2.5 For every agent i, formula p, if $\vdash_X p$ then $\vdash Kip$. □

The conveerse is also true, but the proof is not given in this paper.

Theorem 2.6 *Suppose T is a theory and $T = Cons_X(T)$. For any agent $i \in Ag$, let $T/Ki = \{p|Kip \in T\}$, then $T/Ki = Cons_X(T/Ki)$.* □

Proof:

Suppose $p \in Cons_X(T/Ki)$. According to the compactness and deductive theory, there are some formulas $\{p1, ..., pn\} \subseteq T/Ki$, such that $\vdash_X (p1 \rightarrow (... \rightarrow (pn \rightarrow p)...))$. According to Corollary 2.5 and Lemma 2.1, we get $\vdash_X (Kip1 \rightarrow (... \rightarrow (Kipn \rightarrow Kip)...))$. Since $\{Kip1, ..., Kipn\} \subseteq T$, $T = Cons_X(T)$, so $Kip \in T$. So $p \in T/Ki$.

This theorem shows that every agent's knowledge is logical closed. That is, every agent in logic X has the same inference ability as X.

Corollary 2.7 Suppose p1, ..., pn, q are well-formed formulas, $i_1, ..., i_k$ are agents. If $p1, ..., pn \vdash_X q$, then $K_{i_1}...K_{i_k}p1, ..., K_{i_1}...K_{i_k}pn \vdash_X K_{i_1}...K_{i_k}q$. □

Proof:

Suppose $T = Cons_X(\{K_{i_1}...K_{i_k}p1, ..., K_{i_1}...K_{i_k}pn\})$, $T' = T/K_{i_1}.../K_{i_k}$. Since $\{p1, ..., pn\} \subseteq T'$, $p1, ..., pn \vdash_X q$. So, according to Theorem 2.6, $q \in T'$. Hence, $K_{i_1}...K_{i_k}q \in T$. So $K_{i_1}...K_{i_k}p1, ..., K_{i_1}...K_{i_k}pn \vdash_X K_{i_1}...K_{i_k}q$.

From this corollary, if q is a propositional logical consequence of formulas p1, ..., pn, then the above statement does also hold.

Corollary 2.8 Suppose T is a theory, p is a formula, if $T \nvdash p$ then $T \bigcup \{\neg p\}$ is consistent.

Proof:

Suppose $T \bigcup \{\neg p\}$ is not consistent, then there must be a formula q, such that $T \bigcup \{\neg p\} \vdash_X \neg (q \to q)$.

By applying the compactness theory, we get $T \vdash_X \neg p \to (\neg (q \to q))$, so $T \vdash_X (q \to q) \to p$. So we get $T \vdash_X p$, which contradicts the assumption $T \nvdash_X p$. So, $T \bigcup \{\neg p\}$ must be consistent. □

Definition 2.6 Suppose P is a modal formula. We define P^*, P's $*$-translation, as a formula that contains no modal operator. P^* is defined inductively as follows:

1. If P is a basic formula, then P^*=P.
2. $(P \to Q)^* = (P^* \to Q^*)$
3. $(\neg P)^* = \neg P^*$.
4. $(KiP)^* = P^*$ □

For example, suppose p, q are two basic formulas, then $((Ki(p \to q) \to (Kip \to Kiq))^* = (p \to q) \to (p \to q))$.

Lemma 2.3 Suppose formula $P \in Cons_X(\{\})$, then P^* is a tautology. □

This lemma depends on the fact that every axiom's $*$-translation is a tautology.

Theorem 2.9 *For every $X \in \{W, W4, W5\}$, X is consistent*

Proof:

If X is not consistent, then there must be a formula p such that $\vdash_X \neg (p \to p)$.

According to above lemma, $\neg (p* \to p*)$ must be a tautology. This is a contradiction, so X is consistent. □

3 Model Theory about X: X-Kripke Possible World Structure

In this section, we study the possible world structure for the multi-agent logic class $X \in \{W, W4, W5\}$.

Definition 3.1 W-Kripke Structure

Suppose L is a language based on At and Ag. $\kappa = (W, \pi, w0, R0, R1, ..., Rn)$ is a Kripke structure based on L, where W is an non-empty set, called the world set. $w0 \in W$ is called an initial world; π is a map from W to the subset of At; R0,R1,...,Rn are relations on W. Say structure κ is a W-Kripke structure, if κ satisfies the following four conditions:

1. Every Ri $(i = 0, 1, ..., n)$ is transitive;
2. For every $i = 1, ..., n$, $Ri \subseteq R0$;
3. R0 is reflexive;
4. Every Ri is serial. That is, for every world $w \in W$, every agent $i \in Ag$, the set $\{w' | (w, w') \in Ri\}$ is not empty. □

Generally, we denote an id for the reflexive relation on W, $id = \{(w, w) | w \in W\}$.

Definition 3.2 W4, W5 Kripke Structure

Say W-Kripke structure $\kappa =< W, \pi, w0, R0, ..., Rn >$ is a W4-Kripke Structure, if for every $i \in Ag$, Ri is reflective;

Say W4-Kripke structure $\kappa =< W, \pi, w0, R0, ..., Rn >$ is a W5-Kripke Structure, if for every $i \in Ag$, Ri is symmetric □

Definition 3.3 Suppose $\kappa = (W, \pi, w0, R0, R1, ..., Rn)$ is a X-Kripke structure. We define the semantics entailement relation $\kappa, w \models_X p$, as follows:

1. If $p \in At$, then $\kappa, w \models_X p$ iff $p \in \pi(w)$
2. $\kappa, w \models_X \neg p$ iff $\kappa, w \not\models_X p$
3. $\kappa, w \models_X p \rightarrow q$ iff if $\kappa, w \not\models_X p$ or $\kappa, w \models_X q$
4. For every $i \in Ag$, $\kappa, w \models_X Kip$ iff for every $w' \in W$, if $(w, w') \in Ri$, then $\kappa, w' \models_X p$ □

Definition 3.4 Suppose T is a theory, p is a formula, $\kappa = (W, \pi, w0, R0, R1, ..., Rn)$ is a X-Kripke Structure, then

Say formula p is valid in X-Kripke structure κ, denoted by $\kappa \models_X p$, if $\kappa, w0 \models_X p$;

Say theory T is valid in X-Kripke structure κ, denoted by $\kappa \models_X T$ if, for every formula $p \in T$, p is valid in X-Kripke structure κ;

Say formula p is a semantic entailment of theory T under X logic, denoted by $T \models_X p$ if, for every X-Kripke structure κ, if $\kappa \models_X T$ then $\kappa \models_X p$.

We denote the set of all the semantic entailments of theory T by $Th_X(T)$. □

Proposition 3.1 Every axiom in X is valid in every X-Kripke Structure.

Proof:

Suppose κ is a X-Kripke structure, and w0 is the initial world.

Obviously, axiom A1, $K0p$ is valid in every X-Kripke Structure when p is a tautology.

We can prove that axiom A2, $(K0(K0p \rightarrow K0Kip)$ is valid.

That is κ, $w0 \models_X K0(K0p \rightarrow K0Kip)$. For every $w1$, if $(w0, w1) \in R0$, we should prove κ, $w1 \models_X (K0p \rightarrow K0Kip)$. Suppose κ, $w1 \models_X K0p$. We should prove κ, $w1 \models_X K0Kip$.

First we have, for every $w2$, if $(w1, w2) \in R0$, then κ, $w2 \models_X p$. Now we need to prove κ, $w2 \models_X Kip$. That is, for every $w3$, if $(w2, w3) \in Ri$, then κ, $w3 \models_X p$. Because $Ri \subset R0$, R0 is transitive, so κ, $w3 \models_X p$, so κ, $w1 \models_X K0Kip$. So, $\kappa, w1 \models_X (K0p \rightarrow K0Kip)$. Hence $\kappa, w0 \models_X K0(K0p \rightarrow K0Kip)$.

A3, $K0(Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$, is valid in X-Kripke Structure κ.

$\kappa, w0 \models_X K0(Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$. For every $w1 \in W$, if $(w0, w1) \in R0$, then we should prove $\kappa, w1 \models_X (Ki(p \rightarrow q) \rightarrow (Kip \rightarrow Kiq))$. Suppose $\kappa, w1 \models_X Ki(p \rightarrow q)$ (assumption 1), we should prove $\kappa, w1 \models_X Kip \rightarrow Kiq$. Suppose $\kappa, w1 \models_X Kip$ (assumption), we should prove $\kappa, w1 \models_X Kiq$. That is for every $w2 \in W$, if $(w1, w2) \in Ri$, then $\kappa, w2 \models_X q$. From assumption 1, we have $\kappa, w2 \models_X p \rightarrow q$, from assumption 2, we have $\kappa, w2 \models_X p$, so we have $\kappa, w2 \models_X q$.

So A3 is valid in any X-Kripke Structure.

A4, $K0(K0p \rightarrow p)$, is valid because R0 is reflexive.

A5, $K0(Kip \rightarrow KiKip)$ is valid because every relation $Ri, i = 0, 1, ..., n$ is transitive.

A6, $K0(Ki\neg p \rightarrow \neg Kip)$ is valid because every Ri is serial.

A7, $K0(Kip \rightarrow p)$ is valid in X-Kripke Structure $(X \in \{S4, S5\})$ because Ri is reflective in κ.

A8, $K0(\neg Kip \rightarrow Ki\neg Kip)$ is valid in S5-Kripke Structure κ because Ri is symmetric. □

Proposition 3.2 X's inference rules are safe.

Proof:

Suppose p, $p \rightarrow q$ are true in the X-Kripke structure κ, $\kappa, w0 \models_X p$ and $\kappa, w0 \models_X p \rightarrow q$. Then $\kappa, w0 \models_X q$

Suppose $\kappa, w0 \models_X K0p$. Since $(w0, w0) \in R0$, so $\kappa, w0 \models_X p$. □

From the above two propositions, we can consequently derive X's soundness.

Lemma 3.1 Suppose T is a theory, p is a formula, if $T \vdash_X p$ then $T \models_X p$. □

Theorem 3.3 *Soundness of X.*

1. *$Cons_X(\{\})$ is valid. That is, $Cons_X(\{\}) \subseteq Th_X(\{\})$.*
2. *For every theory T, we have $Cons_X(T) \subseteq Th_X(T)$* □

Can we have completeness of X? That is, $Cons_X(\{\}) = Th_X(\{\})$ and for every theory T, $Cons_X(T) = Th_X(T)$. From the above discussion, we already have $Cons_X(T) \subseteq Th_X(T)$. In the next section, therefore, we will prove $Cons_X(T) \supseteq Th_X(T)$.

4 X's Completeness

In this section, we will prove that the X logic system is complete. The proof demands the application of some special techniques. First, we present the following concepts.

Definition 4.1 Say theory T is X-complete, if $T = Cons_X(T)$ and for every formula $p \in L$, either $p \in T$ or $\neg p \in T$. Obviously, L is a X-complete theory. □

Theorem 4.1 *Suppose T is a theory, then*

1. *If T is contradiction in X, then $Cons_X(T)$ is X-complete.*
2. *If T is consistent in X, then T must have a consistent X-complete superset theory.*
3. *If T is consistent X-complete , then for every agent i, there must be a consistent X-complete set T' such that $T/Ki \subseteq T'$.* □

Proof:

1. Since T is a contradiction in X, there must be a formula p such that $T \vdash_X \neg(p \rightarrow p)$. Since every formula q is a logical consequence of $\neg(p \rightarrow p)$, $Cons_X(T) = L$ must be X-complete.

2. Suppose T is consistent. Now we prove that T has a consistent X-complete superset T'. T' is constructed as follows:

   Suppose p1, p2,... is the enumeration of all the formulas in L.

   T_0=T, for every $i \geq 0$, we define

   $T_{i+1} = Cons_X(T_i)$ if $pi \in Cons_X(T_i)$ or $\neg pi \in Cons_X(T_i)$

   $T_{i+1} = Cons_X(T_i) \cup \{\neg pi\}$ else

   Suppose $T' = \bigcup_{i=0}^{i=\infty} T_i$, then it is easy to prove that T' is a consistent X-complete superset of T.

3. Suppose $T1 = T/Ki$. It is easy to prove that T1 is also consistent.

   If T1 is not consistent, then there must be some formulas $\{p1, ..., pn\} \in T1$ such that $\{p1, ..., pn\} \vdash_X \neg(p \rightarrow p)$. So, we can prove that $\vdash_X (Kip1 \rightarrow ... \rightarrow (Kipn \rightarrow Ki(\neg(p \rightarrow p))...)$. Since $Ki(\neg(p \rightarrow p)) \vdash_X \neg Ki(p \rightarrow p)$, we have $\vdash_X \neg Kip1 \vee ... \vee \neg Kipn$. Notice that $Kip1 \in T, ..., Kipn \in T$, T is a contradiction. This is a contradiction to the assumption. So, T1 is consistent in X.

   According to item 2 of this corollary, we can conclude that T1 has a consistent X-complete superset T', such that $T/Ki \subseteq T'$.

Corollary 4.2 Suppose T is a consistent theory in X. Then, for any formula p, if $p \notin Cons_X(T)$, T must have a consistent X-complete superset T' such that $\neg p \in T'$.

Hint: Supposing that the formula p1 in the enumerated sequence of the above theorem is q, then we can obtain this corollary. □

Now, we can construct the Canonical X-Kripke structure, based on consistency theory T in X, as follows:

Definition 4.2 Canonical X-Kripke structure over a X-consistent theory T.

Suppose T is an X-consistent theory. We construct the Canonical X-Kripke structure $\kappa = (W, \pi, w0, R0, ..., Rn)$ as follows:

1. $W = \{T' | T'$ is the consistent X-complete set $\}$, and $w0 \in W$, is a consistent X-complete superset of T.

2. For every $w \in W$, we define $\pi(w) = \{p | p \in At$ and $p \in w\}$

3. For every $w1 \in W, w2 \in W$, $(w1, w2) \in Ri$ iff $w1/Ki \subseteq w2$. □

Then it is easy to prove:

Proposition 4.3 Every Canonical X-Kripke structure $\kappa = < W, \pi, w0, R0, R1, ..., Rn >$ is a X-Kripke structure.

Proof:

1. Every Ri is transitive.

   If $\{(w1, w2), (w2, w3)\} \subseteq Ri$, then we should be able to prove $(w1, w3) \in Ri$. For every $Kip \in w1$, since $w1 = Cons_X(w1)$, so $KiKip \in w1$. Since $(w1, w2) \in Ri$, so $Kip \in w2$. Since $(w2, w3) \in Ri$, so $p \in w3$. So $(w1, w3) \in Ri$.

2. For agent $i \in Ag$, $Ri \subseteq R0$.

   Obviously, if $(w1, w2) \in Ri$ then, for every formula p, if $Kip \in w1$, then $p \in w2$. Now, we can prove that if $K0p \in w1$ then $p \in w2$. Since $K0p \in w1$ and $w1 = Cons_W(w1)$, so $Kip \in w1$, so $p \in w2$. So Ri is a subset of R0.

3. R0 is reflexive.

   This is obvious, for every $w \in W$, if $K0p \in w$, since $w = Cons_X(w)$, then $p \in w$. So $(w, w) \in R0$.

4. For every agent i and every world w, the set $\{w1 | (w, w1) \in Ri\}$ is not empty.

   This is true according to the Theorem 4.1 items 3.

5. When $X \in \{S4, S5\}$, then every Ri is reflective.

   For every $w \in W$, $w = Conx_X(w)$. If $Kip \in w$, then p has a proof in w. The proof is Kip, $K0(Kip \rightarrow p)$, $Kip \rightarrow p$, p. So $p \in w$. So $(w, w) \in Ri$.

6. When $X = S5$, then every Ri is symmetric.

   Suppose $w1 \in W, w2 \in W$, $(w1, w2) \in Ri$. Now we prove $(w2, w1) \in Ri$. If not, then there is a formula p such that $Kip \in w2$, $p \notin w1$. Since w1 is consistent S5-complete, so $\neg p \in w1$, $\neg Kip \in w1$. According the axiom 8, we have $Ki\neg Kip \in w1$. Since $(w1, w2) \in Ri$, $\neg Kip \in w2$, and w2 is inconsistent. This is a contradiction. So $(w2, w1) \in Ri$.

□

Lemma 4.1 Suppose $w \in W$, and p is a formula. If $Kip \notin w$, then there must be a $w' \in W$, such that $(w, w') \in Ri$ and $\neg p \in w'$.

Proof:

First according to theorem 4.1 items 3, $T1 = \{q | Kiq \in w\}$is consistent. According to theorem 2.6, T1 is closed. Since $Kip \notin w$, w is closed, so $p \notin T1$. According to Corollary 4.1 items 2, T1 has a consistent X-complete superset w', such that $\neg p \in w'$ and $(w, w') \in Ri$.

□

Now, it is easy to prove our main statement:

Theorem 4.4 For every formula $p \in L$, $\kappa, w \models p$ iff $p \in w$. □

This proof is based on the induction of formula p's length.

1. If $p \in At$, then it is obvious that $\kappa, w \models p$ iff $p \in w$.

2. Suppose the above statement is true for every formula p whose length is not greater than t.

3. Suppose p is a formula whose length is greater than t. Then, we can prove the above statement by following situations.

   (a) p is $\neg q$, where q's length is not greater than t. Then,
   $\kappa, w \models_X p$ iff $\kappa, w \not\models_X q$ iff $q \notin w$ iff $\neg q \in w$ iff $p \in w$.

   (b) p is $q \rightarrow r$, where both formula q and r's length are not greater than t.
   $\kappa, w \models_X p$ iff $\kappa, w \models_X q \rightarrow r$ iff $\kappa, w \not\models_X q$ or $\kappa, w \models_X r$ iff $q \notin w$ or $r \in w$ iff $\neg q \in w$ or $r \in w$ iff $q \rightarrow r \in w$ iff $p \in w$.

   (c) p is Kiq, where q's length is not greater than t. Suppose $\kappa, w \models_X Kiq$. Then for every $w' \in W$, if $(w, w') \in Ri$ then $q \in w'$. Now, we prove $Kiq \in w$. If $Kiq \notin w$ then, according to Lemma 4.1, there must be a consistent complete superset w', such that $(w, w') \in Ri$ and $\neg q \in w'$. This is a contradiction, so $Kiq \in w$.
   On the other hand, suppose $Kiq \in w$. For every w', if $(w, w') \in Ri$, it is obvious that $q \in w'$. According to the induction step, $\kappa, w' \models_X q$. Hence, $\kappa, w \models_X Kiq$. That is $\kappa, w \models_X p$

Theorem 4.5 *Suppose T is a consistent theory. Then, for every Canonical X-Kripke structure of T, $\kappa = (W, \sigma, w0, R0, ..., Rn)$ is a X-Kripke model of T.*

Proof:

First by proposition 4.3, κ is a X-Kripke structure.

Notice, Since $T \subseteq w0$ and w0 is a consistent X-complete superset of T, so by theorem 4.4, we have $\kappa, w0 \models_X T$. □

According to corollary 4.2 and theorem 4.5, we get:

Corollary 4.6 If $p \notin Cons_X(T)$, then we can choose an initial world w0 for the Canonical X-Kripke structure of T, such that $\kappa, w0 \models_X \neg p$ □

Suppose theory T is a X-consistent theory, $Can_X(T)$ is the set of all the Canonical X-Kripke structures of T. Obviously, it is a subset of the models of T. From corollary 4.3 and theorem 4.5, it is easy to see that $Cons_X(T) = \{p|p$ is valid in all structures of $Can_X(T)\}$. So, we get the following theorem:

Theorem 4.7 *Completeness of X.*

Suppose T is a X-consistent theory. Then, all the semantic entailment of T is the consequence conclusion of T. In other words, $Th_X(T) \subseteq Cons_X(T)$. □

Theorem 4.8 *Complete Theorem*

1. *Formula p is X-consistent iff p has a X-Kripke structure.*
2. *For every X-consistency theory T,* $Th_X(T) = Cons_X(T)$. □

5 Traditional Multi-agent Knowledge Logic System S5, S4, KD4.

In this section, we briefly introduce the traditional knowledge systems S5, S4, KD4. In the next section, we will discuss the relationship between our X logic systems and logical systems proposed here.

Notice that there is no fool reasoner in traditional multi-agent logic systems. So we will discuss the language L1 based only on normal agent set $Ag1 = Ag - \{0\} = \{1, ..., n\}$ and propositional set At. Obviously, L1 is a subclass of the language L which we discussed in section 2.

Definition 5.1 [The Traditional Multi-agent Knowledge system S5].

For every L1's formulas p, q, agent $i \in Ag1$.

S4's Axioms:

AS1: p, if p is a tautology.

AS2: $Ki(p \to q) \to (Kip \to Kiq)$

AS3: $Kip \to KiKip$

AS4: $Ki\neg p \to \neg Kip$

AS5: $Kip \to p$

AS6: $\neg Kip \to Ki\neg Kip$.

S5's Inference Rules:

Modus Ponens: p, $p \to q \Rightarrow q$

Necessitation Rule: $p \Rightarrow Kip$. □

Generally AS5 is called knowledge axiom or T-axiom, AS2 is called K-axiom, or distributed axiom, AS3 is called positive introspective axiom or 4-axiom, AS4 is called D-axiom, AS6 is called negative introspective axiom or 5-axiom.

Definition 5.2 S4, KD4

S4 is the logic system S5 without axiom AS6.

KD4 is the S4 logic system deleting the axioms AS5. □

Suppose $Y \in \{KD4, S4, S5\}$, we define the proof relationship between a theory T and formula q as in [2], and denoted as $T \vdash_Y q$, and denote $Cons_Y(T)$ as $\{q | T \vdash_Y q\}$.

Definition 5.3 Y-Kripke-structure

Say Kripke-structure $\kappa =< W, \pi, R1, ..., Rn >$ is a Y-Kripke structure, if

1. $W \neq \{\}$, W is called a possible world set.
2. π is a map from W to 2^{At}.
3. R1, ..., Rn are relations on W such that

1. Every Ri is transitive and serial.
2. Every Ri is reflective if $Y \in \{S4, S5\}$.
3. Every Ri is symmetric if $Y = S5$.

□

Definition 5.4 Suppose $\kappa =< W, \pi, R1, ..., Rn >$ is a Y-Kripke structure. For every formula $q \in L1$, every world $w \in W$, we define $\kappa, w \models_Y q$ as follows:

1. $\kappa, w \models_Y q$ iff $q \in \pi(w)$, if $q \in At$.
2. $\kappa, w \models_Y \neg q$ iff $\kappa, w \not\models_Y q$.
3. $\kappa, w \models_Y p \rightarrow q$ iff $\kappa, w \not\models_Y p$ or $\kappa, w \models_Y q$.
4. For every $i \in Ag1$, $\kappa, w \models_Y Kip$ iff for every $w' \in W$, if $(w, w') \in Ri$ then $\kappa, w' \models_Y p$ □

Definition 5.5 Suppose $\kappa =< W, \pi, R1, ..., Rn >$ is a Y-Kripke structure, T is a theory, p is a formula:

Say p is valid in κ, denoted by $\kappa \models_Y p$, if for every world $w \in W$, $\kappa, w \models_Y p$.

Say T is valid in κ, denoted by $\kappa \models_Y T$, if for every formula $p \in T$, $\kappa \models_Y p$.

Say p is a Y-entailment consequence of T, denoted by $T \models_Y p$, if for every Y-Kripke structure κ, if T is valid in κ, then p is also valid in κ. We denote all the Y-entailment consequence of T is $Th_Y(T)$. □

Theorem 5.1 *For every theory S and formula q in L1*

1. $S \models_{KD4} q$ *iff* $S \vdash_{KD4} q$
2. $S \models_{S4} q$ *iff* $S \vdash_{S4} q$
3. $S \models_{S5} q$ *iff* $S \vdash_{S5} q$ $\square$

Modal logic Y does not have the deductive properties. For example, $p \vdash_Y Kip$, but $\not\vdash_Y (p \rightarrow Kip)$. In the following section, we will study the relationship between class X and class Y.

Briefly, the main difference between class X and class Y are:

1. Logic class Y has Necessitation Inference Rule: $p \Rightarrow Kip$. But X does not have this inference rule, X has only Safeness Rule $K0p \Rightarrow p$.

2. X has a fool reasoner, but Y does not.

3. Every axiom of X can be viewed as the fool's common knowledge, but axioms of Y can only be viewed as agent's knowledge.

4. In X, the knowledge axiom only holds for fool reasoner, that is only $K0p \rightarrow p$ hold in X. In S4 and S5, the knowledge axiom holds for every agent. Of course in KD4, no agent has the knowledge axiom.

5. X has a good computational property, the deduction properties [Ref Theorem 2.4], but Y does not have this property [ref [2]].

6 Relationship between Class X and Class Y

Suppose f is a map $f : \{W, W4, W5\} \rightarrow \{KD4, S4, S5\}$, such that $f(W) = KD4, f(W4) = S4, f(W5) = S5$. In this section, we will study the relationship between X and $f(X)$.

Theorem 6.1 *For every theory T of L1, and formula q of L1, if* $S \models_X q$ *then* $S \models_{f(X)} q$.

Proof:

Suppose the above statement is not true. Then there are some theory S and formula q on L1 such that $S \models_X q$ *but* $S \not\models_{f(X)} q$.

Since $S \not\models_{f(X)} q$, *there must be a* $f(X)$*-Kripke-Structure* $\kappa =< W, \pi, R1, ..., Rn >$ *such that* $\kappa \models_{f(X)} S$ *but* $\kappa \not\models_{f(X)} q$. *So there must be a world* $w' \in W$ *such that* $\kappa, w' \models_{f(X)} \neg q$.

Now, we construct a X-Kripke-Structure $\kappa_1 =< W_1, w0, \pi_1, R0_1, R1_1, ..., Rn_1 >$ *as following:*

1. $W_1 = W$, $w0 = w'$, $\pi_1 = \pi$, *For i=1,...,n,* $Ri_1 = Ri$.

2. $R0_1 = trans(id \cup R1 \cup ..., \cup Rn)$ [1] *if* $X \in \{W, W4\}$.

3. $R0_1 =$ *trans-symmetric* $(id \cup R1 \cup ..., \cup Rn)$ [2] *if* $X = W5$.

It is easy to check that κ_1 *is a X-Kripke-Structure.*

Inductively on formula's length, we can prove that:

Lemma 6.1 For every formula q in L1, every world $w \in W$, $\kappa, w \models_{f(X)} q$ **iff** $\kappa_1, w \models_X q$. □

If q is an atom, then it is obviously true.

Suppose the above statement is true for the formulas whose length is not greater than t.

If q is $\neg p$, *and p's length is not greater than t, then for every world* $w \in W$, $\kappa, w \models_{f(X)} q$ *iff* $\kappa, w \not\models_{f(X)} p$ *iff* $w \in W$, $\kappa_1, w \not\models_X p$ *iff* $\kappa_1, w \models_X q$.

If q is $p1 \rightarrow p2$, $p1, p2$*'s length is not greater than t, then it is also easy to prove the above statement.*

If q is Kip, then for every world $w \in W$, $\kappa, w \models_{f(X)} q$ *iff for every* $(w, w') \in Ri$, $\kappa, w' \models_{f(X)} p$ *iff for every world* $(w, w') \in Ri$, $\kappa_1, w' \models_X p$ *iff* $\kappa_1, w \models_X q$.

So we have inductively proved our main statement.

Since $\kappa, w' \models_{f(X)} \neg q$ *and* $\kappa \models_{f(X)} S$, *so* $\kappa 1, w0 \models_X \neg q$ *and* $\kappa 1, w0 \models_X S$.

Since $S \models_X q$ *and* $\kappa 1, w0 \models_X S$, *so* $\kappa 1, w0 \models_X q$.

Obviously it is a contradiction and hence we prove our theorem. □

Corollary 6.2 For every theory S and formula q on L1, if $S \vdash_X q$ then $S \vdash_{f(X)} q$. □

Theorem 6.3 *For every theory S and formula q on L1, if we have* $S \models_{f(X)} q$ *then* $K0S \models_X K0q$.

Proof:

Suppose there are theory S and formulas q on L1 such that $S \models_{f(X)} q$ *and* $K0S \not\models_X K0q$.

We can find a X-Kripke structure $\kappa =< W, \pi, w0, R0, R1, ..., Rn >$ *such that:*

1. For every $\kappa, w0 \models_X K0S$.

2. $\kappa, w0 \not\models_X K0q$.

Suppose $W_1 = \{w | (w0, w) \in R0\}$. *Then we can find that:*

3. For every $w \in W_1$, $\kappa, w \models_X S$.

[1] trans(S) is the least transitive relation containing the relationship S

[2] trans-symmetric(S) is the least transitive and symmetric relation containing the relationship S

4. There is a $w' \in W_1$*, such that* $\kappa, w' \not\models_X p$.

Now we construct a $f(X)$*-Kripke Structure* $\kappa_1 =< W_1, \pi_1, R1_1, ..., Rn_1 >$ *such that* $\kappa_1 \models_{f(X)} S$ *and* $\kappa_1 \not\models_{f(X)} q$.

$f(X)$*-Kripke-Structure* $\kappa_1 =< W_1, \pi_1, R1_1, ..., Rn_1 >$ *is constructed as follows:*

1. W_1 *is defined above.*

2. $\pi_1 = \pi$

3. For every $i = 1, ..., n$, $Ri_1 = Ri \cap (W_1 \times W_1)$.

Obviously, kappa$_1$ *is a f(X)-Kripke structure.*

It is easy to prove (by inductive on formula's length) that:

For every formula $p \in L1$ **and** $w \in W_1$, $\kappa, w \models_X p$ **iff** $\kappa_1, w \models_{f(X)} p$.

For every $w \in W$, $\kappa, w \models_X S$*, we have* $\kappa_1 \models_{f(X)} S$*; There is a* $w' \in W_1$, $\kappa, w' \not\models_X q$*, we have* $\kappa_1 \not\models_{f(X)} q$*. Since* $S \models_{f(X)} q$*, from* $\kappa_1 \models_{f(X)} S$*, we get* $\kappa_1 \models_{f(X)} q$*. This is a contradiction to kappa*$_1 \not\models_{f(X)} q$*. And hence we prove our theorem.* □

Then we get the most important conclusion:

Theorem 6.4 *for every theory S1, formula p of L1, we have*

1. If $S \vdash_{f(X)} q$ *then* $K0S \vdash_X K0q$.

2. If $S \vdash_X q$ *then* $S \vdash_{f(X)} q$. □

7 Conclusion

From above discussion, we have seen that traditional knowledge and belief modal logic systems about multi-agent system only reflect our fool's inference ability. Our new logic class has more advantages than traditional logic. First is the inductive property. Second is that it has no necessary inference rule. So real world knowledge may be not known by any agent. Third, every normal agent's knowledge can be inconsistent with the real world knowledge. For example, suppose i is a normal agent, then $p \wedge Ki \neg p$ is valid in W, $p \wedge \neg Kip$ is valid in W4, W5. Fourth, it gives an especially complete account of what is common knowledge and how to use common knowledge. In contract to the research in [6] and [5], where they try to find what is common knowledge in traditional S5 systems, we only describe what is common knowledge, and then we concentrate our attention on how to use it.

There are fundamental relationships between our logic class and the traditional class. Our logic class is much better than the traditional logic, since traditional logic can be expressed in our new logic. Logic proposed in this paper correctly models the multi-agent reasoning system based on shared common knowledge view. It can solve most of the problems in this field. Some examples, such as the conway paradox, can be found in [23], [27]. In further upcoming papers, we will describe our logic's proof theory, the common knowledge concept here and the concept in [1] [5], and the least information extension problem which was discussed in [5].

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