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A Connotative Treatment of Circumscription (Preliminary Report)

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(PRELIMINARY REPORT)

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Abstract

Circumscription proposed by McCarthy is one of the most hopeful formalizations of nonmonotonic aspects of commonsense reasoning. It has several versions, however, they are all proposed for denotative minimization of predicates, that is, circumscription minimizes the extension of predicates. Regarding such treatment, this paper considers three problems; absence of abnormal things, a limitation on equality and formalization of the unique name assumption. This paper proposes a solution for them by presenting a connotative treatment of circumscription. This treatment is based on the idea of circumscribing predicates connotatively, that is, minimizing the set of names denoting objects which satisfy a certain predicate.

1 Introduction

Consider the following situation.

Situation: "A person in a living room hears someone knocking on the front door. He knows that if someone knocks it is normally a man, however, he remembers that one time there was an exception: Tweety, a woodpecker, knocked on the door. He is very tired, so rather than go to the door he calls 'Who is it?'. The visitor does not answer..."

In this situation, we expect that he would conclude that the anonym is a man after all and would reluctantly walk to the door. He knows that if someone knocks it is normally a man and he has not received any information that contradicts the conclusion. McCarthy proposed a way to represent facts about what is "normally" the case. The key idea is minimization of abnormality and, to minimize some predicates, he propose a form called circumscription [5, 6]. For instance, the situation is expressed as follows. (To clarify our arguments, it is simplified.)

Example 1. Let a sentence, A, be

$$\forall x(Knocks(x) \wedge \neg Ab(x) \supset Man(x))$$
$$\wedge Knocks(Anonym)$$
$$\wedge Ab(Tweety)$$
$$\wedge \neg Man(Tweety)$$

and, according to the idea of circumscription, we minimize (circumscribe) a predicate, Ab, with allowing a predicate, Man, to vary. We do so because we want to think that each object is normal unless available information shows otherwise, and because we want to know whether the object is a man or not. However, there are three unsatisfied points on such treatment of circumscription.

1) Absence of abnormal things

The circumscription of Ab with allowing Man in A to vary yields

$$\forall x(Ab(x) \equiv x = Tweety).$$

This circumscription says that *Tweety* is the only abnormal. This result is too strong and somewhat unnecessary. What we want is the conclusion that the anonym is a man, that is, we want simply to think that the anonym is not abnormal if it is consistent with the given sentence; we do not need to know what is abnormal. To obtain the intended conclusion, why do we have to think that only *Tweety* satisfies the property, *Ab*? It seems more natural to think that abnormal things except Tweety would still exist.

2) A limitation on equality

The most important and serious problem concerns a limitation of circumscription on equality. We showed that circumscription (of any predicates with any predicates or functions that are allowed to vary) cannot yield a new fact that varies the least cardinal number of domains of models of a given sentence if there exists a model whose domain consists of finite objects [1]. Returning to Example 1, from A and the result of circumscription (the above sentence) we can obtain

$$Anonym \neq Tweety \supset Man(Anonym).$$

However, we can never obtain the expected fact that the anonym is a man, $Man(Anonym)$. Because if circumscription could yield the fact then the circumscription changes the least cardinality from 1 to 2. (Note that there is a model of A whose domain consists of one object; so the least cardinality of A is 1. However, in a model of A where $Man(Anonym)$ holds, $Anonym \neq Tweety$ holds, too. So the least cardinal number of such models is 2). This is a contradiction and shows that without additional axioms to A circumscription could never yield our intended results.

3) Formalization of the unique name assumption

Readers may think of using the unique name assumption for the above problem. However, we do not presently know satisfiable formalization of the unique name assumptions. This is closely related to the limitation mentioned above.

Reiter introduced the idea of the "unique name assumption (or hypothesis)" [7], that is, distinct names denote distinct objects unless available facts imply that those objects are equal. Some approaches to formalization of the unique name assumption have been taken, but, we are not satisfied with them yet. For instance, McCarthy's solution [6] uses the language which involves the names themselves as the only objects. So all assertions about objects must be expressed as assertions about names. This may be considered unnatural [4]. Lifschitz presented another solution [4], which provides (finitely many) symbols for both names and denotation and introduced a unary function from names to objects. His solution also involves axioms which represent that the names are distinct from each other. Notice that these axioms provides a sufficient number of objects to yield facts on inequality under the limitation mentioned in 2). Yet their solutions are both insufficient by reason of the following two points; i) they do not express the unique name assumption for infinite names (for the infinite names, infinite axioms would be needed.) and ii) if the given axioms involves the domain closure axiom their solutions cannot be applied generally. Given a sentence with a domain closure axiom,

$$\forall x(x = Jekyll \vee x = Hyde) \wedge Man(Stevenson),$$

their solutions require three distinct names for *Jekyll*, *Hyde* and *Stevenson* which means that the domain of any models of (inconsistent) sentences involving such objects must consist of at least three objects. Of course, it is impossible by the domain closure axiom, which asserts at most two objects.

This paper attempts to solve these problems. Our approach is to present a way to circumscribe predicates connotatively, whereas each version of circumscription proposed so far circumscribes predicates denotatively. Our result is called the connotative circumscription. It minimizes the set of names denoting objects which satisfy certain predicates. Intuitively, the sentence expresses an idea; "the names that can be shown to denote the objects satisfying a certain property P from certain facts A are all the names denoting objects which satisfy P". We show this in the next section.

2 Connotative Circumscription

Let us consider a universe in which objects denote syntactical form of grounded terms, that is, syntactically distinct grounded terms correspond to distinct objects with each other in the universe (in this sense, objects in the universe can be considered as *names* of grounded terms). We call the universe *the connotative universe* and, by contrast, we call the usual universe which is represented by a given sentence *the denotative universe*. Now, let us return back to Example 1. In the connotative universe, $Anonym, Tweety$ are denoted by distinct objects, $\widehat{Anonym}, \widehat{Tweety}$, while, in the denotative, they need not be distinct always. If we minimize abnormality in the connotative universe, that is, if we minimize a set of objects which denote abnormal objects in the denotative universe, we obtain a result, "$\widehat{Tweety}$ is only the name that denotes abnormal thing". $\widehat{Anonym}$ is another name, so we would conclude that "$Anonym$ which is denoted by $\widehat{Anonym}$ is not abnormal". As connotative minimization constaraints only objects which are named and it say nothing about objects with no names, such abnormal things may always exists. This could also be a solution of the first problem in the first section.

In some papers on the unique name assumption, we can see a phrase, to *minimize equality*, however, it does not express correctly what we want, because equality is minimized from its origin denotatively. What we want is to minimize equality connotatively. We want to minimize the case that objects with syntactically distinct forms are identical with each other.

For these reasons, it seems reasonable to use circumscription connotatively. We propose a connotative circumscription, which minimize a certain predicate connotatively. Details are as follows.

Our approach uses the language $L = L_L \cup L_E \cup L_I$. L_L consists of the logical symbols. L_E is the external language which is used for representation of the concerning universe, that is, let a given sentence A be a sentence of $L_L \cup L_E$, and L_I is the internal language which is used for representation of the connotative universe. We assume that a given sentence, A is *closed*, that is, no variable occurs free in A.

L_L consists of parentheses, (,), logical connectives, $\wedge, \vee, \neg, \supset, \equiv$, quantifiers, $\forall, \exists$, and equality, $=$. Both L_E and L_I consist of symbols for predicates (constants and variables) and functions (constants and variables), but are disjoint with each other. (Object constants are considered 0-ary functions.) If a symbol, K, is in L_E, L_I involves a *similar* symbol, $\hat{K}$, represented by putting "ˆ" over K (We say $\mathbf{K}(= K_1, \ldots, K_n)$ *is similar to* $\mathbf{L}(= L_1, \ldots, L_n)$ if for each i K_i and L_i are both predicates (or, functions) and have the same arity). That is, an n-ary predicate constant P is in L_E, L_I involves an n-ary predicate constant $\hat{P}$, and similarly to functions, F, and each variables, p and f ; $P, p, F, f, ...$ are in L_E and $\hat{P}, \hat{p}, \hat{F}, \hat{f}, ...$ are in L_I. Especially, we say a symbol, $\hat{K}$, is *paired* if K is a similar symbol in L_E.

We use symbols starting with a capital letter (or "ˆ" and a capital letter) for constants and a small letter for variables.

Moreover, let L_I involve a unary predicate, $\hat{D}$ (for being a member of the connotative universe) and a binary predicate, $\hat{=}$ (for connotative equality). ($\hat{D}$ is not always paired, and $\hat{=}$ is not paired in the above sense. As we will see soon, $\hat{=}$ is related to equality, $=$, in L_L.) Let L_I involve the following functions: 0 (for zero), S (for successor), $+$ (for addition), $\times$ (for multiplication), and $\uparrow$ (for exponentiation). Let it also involve a unary function Π (for assignment function of objects to its names in connotative universe).

CONNOTATIVIZATION

We handle *the connotative universe* in addition to *the denotative universe* and restrict the application of circumscription to the connotative universe. The connotative universe can be considered as the domain of the inner world. We take a sentence from the outer world into the inner world. We call this treatment *connotativization*.

The connotative universe involves a set of natural numbers, some of which can be considered as names denoting grounded terms in the denotative universe. Especially, with respect to a ceartain tuple of functions, $\mathbf{F}$, in L_E, names are distinctly assigned to *terms in* $\mathbf{F}$, (that is, terms constracted with functions in $\mathbf{F}$ and object variables in L_E). The other functions in L_E need not to be distinguished. Such universe is called *connotative universe w.r.t.* $\mathbf{F}$.

First, to construct such connotative universe, we start with introduction of axioms on *unique naming*. Then, we consider connotativization.

1) Unique Naming. The unique naming embodies each paired function, $\hat{F}$, in $\hat{\mathbf{F}}$, in an adequate function based on the number theory such that syntactically distinct terms in $\mathbf{F}$ can be distinguished each other, where $\hat{\mathbf{F}}$ is a certain tuple of paired functions in L_I and $\mathbf{F}$ is a tuple of functions in L_E which functions in $\hat{\mathbf{F}}$ are paired with. Speaking more concretely, as a result, each grounded term in $\mathbf{F}$ is denoted by a distinct number each other as its name in the connotative universe. Before introducing the axiom, we need some preliminaries.

(a)**Number Theory** Unique naming is based on a number theory represented by the following sentence, N_A, where N_A is a sentence

$$\begin{array}{lll} & \forall\hat{x}(S(\hat{x}) \neq 0) & \wedge\ \forall\hat{x},\hat{y}(S(\hat{x}) = S(\hat{y}) \supset \hat{x} = \hat{y}) \\ \wedge & \forall\hat{x}(\hat{x} + 0 = \hat{x}) & \wedge\ \forall\hat{x},\hat{y}(\hat{x} + S(\hat{y}) = S(\hat{x} + \hat{y})) \\ \wedge & \forall\hat{x}(\hat{x} \times 0 = 0) & \wedge\ \forall\hat{x},\hat{y}(\hat{x} \times S(\hat{y}) = (\hat{x} \times \hat{y}) + \hat{x}) \\ \wedge & \forall\hat{x}(\hat{x}\uparrow 0 = S(0)) & \wedge\ \forall\hat{x},\hat{y}(\hat{x}\uparrow S(\hat{y}) = (\hat{x}\uparrow\hat{y}) \times \hat{x}), \end{array}$$

where $x \neq y$ stands for $\neg(x = y)$.

In the remainder of this paper, we abbreviate $S(0)$ as 1, $S(S(0))$ as 2, and so on. We also abbreviate $\hat{x}\uparrow\hat{y}$ as $\hat{x}^{\hat{y}}$. We call $0, 1, \ldots$ (natural) numbers, and the set of numbers is denoted by $\hat{N}$.

Then, we introduce a function, $<>^n$, which plays a main role in unique naming.

For each $n \leq 1$, let a n-ary recursive function ,$<>^n$, satisfy the following conditions;

$<>^n$: $\hat{N}^n \to \hat{N}$, and for all numbers $a_1, \ldots, a_n, b$, there is a 2-ary recursive function, β , such that if $< a_1, \ldots, a_n >^n = b$, $\quad \beta(b, 0) = n$ and $\beta(b, i) = a_i \quad (1 \leq i \leq n)$.

It is well-known that such a function, $<>^n$ exists. For example, $< a_1, \ldots, a_n >^n \equiv 2^n \times 3^{a_1} \times \cdots \times Pr(n+1)^{a_n}$, where $Pr(n)$ is the n-th prime number. Of course, we can easily construct a recursive function, β, based on unique factorization. It means that $<>^n$ can be considered as a function which encodes finite sequences of numbers to numbers which it is computable to decode into their original sequences. Note that it also means that distinct sequences are encoded into distinct numbers. We take such a $<>^n$.

b) **Unique Name Axiom.** Then, we introduce the unique name axiom. The unique name axiom, denoted by $U_N(\hat{\mathbf{F}})$, which is the conjunction of sentenses

$$\forall x_1, \ldots, x_n(\hat{F}(x_1, \ldots, x_n) \equiv < \dot{F}, x_1, \ldots, x_n >^{n+1}),$$

for each $\hat{F}$ in $\hat{\mathbf{F}}$, where $\hat{F}$ is a paired function and $\dot{F}$ is a distinctly assigned number to each function, F, in L_E which $\hat{F}$ is paired with.

Note that, as a grounded term in $\mathbf{F}$ is represented by a sequence of numbers, for each grounded term we can obtain a distinct number as its name by applying recursively these functions that correspond to each function in the term. (Also, note that $< \dot{F}, x_1, \ldots, x_n >^{n+1}$ is a function of L_I.) We can consider $< \dot{F}, x_1, \ldots, x_n >^{n+1}$ as an n-ary function from $\hat{N}^n$ to $\hat{N}$, so we abbreviate this as $\hat{F}(x_1, \ldots, x_n)$.

Next, we consider connotativization.

In connotativization, we use an unary predicate constant, $\hat{D}$, in L_I to represent the connotativized outer domain (the denotative universe), that is, $\hat{D}$ stands for the connotative universe. Also, we use an unary function constant, Π, (in L_I) of the connotative universe into the denotative universe. Intuitively, the connotative universe consists of names and Π maps each name to an object denoted by the name.

2) Connotativization of quantifiers. To connotativize quantifiers, we introduce the next sentence

(c) $A^{conn(\Pi,\hat{D})}$,

where $A^{conn(\Pi;\hat{D})}$ is the conjunction of sentences formed by *connotativization of universal quantifier* ($C\forall$) and *connotativization of existential quantifier* ($C\exists$).

Connotativization of universal quantifier is to replace each universal quantifier $\forall x B(x)$ in A by

$$\forall \hat{x}(\hat{D}(\hat{x}) \supset B(\Pi(\hat{x})))$$

and connotativization of existential quantifier is to replace each existential quantifier $\exists x B(x)$ in A by

$$\exists \hat{x}(\hat{D}(\hat{x}) \wedge B(\Pi(\hat{x}))).$$

Connotativization of quantifiers means that objects which are denoted by names in the connotative universe satisfies a given sentence.

3) Connotativization of symbols.

We have inner predicates and functions in L_I corresponding to each outer ones in L_E. Then, we introduce a sentence which relativize them between the both universes in symbols.

(d) **Connotative Function Axiom.** First, we define the paired functions as functions which acts on the connotative universe. The connotative function axiom, denoted by $\hat{F}-axiom(\hat{D})$, is the conjunction of sentences

$$\forall \hat{x}_1, \ldots, \hat{x}_n(\hat{D}(\hat{x}_1) \wedge \ldots \wedge \hat{D}(\hat{x}_n) \supset \hat{D}(\hat{F}(\hat{x}_1, \ldots, \hat{x}_n)))$$

for each n-ary paired function, $\hat{F}$ in L_I (for 0-ary functions, $\hat{F}_c$, $\hat{D}(\hat{F}_c)$),

$\hat{F}-axiom(\hat{D})$ states that each $\hat{F}$ can be considered as a function of $\hat{D}$ into $\hat{D}$ and that the result of application of each paired function to any tuple of objects in the connotative universe is an object in the connotative universe.

(e) **Π-axiom.** We connect symbols and thieir paired symbols with each other by the $\Pi-axiom$, denoted by Π-axiom($\Pi; \hat{D}; \hat{=}$), which is the conjunction of (ΠF) sentences

$$\forall \hat{x}_1, \ldots, \hat{x}_n(\hat{D}(\hat{x}_1) \wedge \ldots \wedge \hat{D}(\hat{x}_n) \supset F(\Pi(\hat{x}_1), \ldots, \Pi(\hat{x}_n)) = \Pi(\hat{F}(\hat{x}_1, \ldots, \hat{x}_n)))$$

for each n-ary function, F, in L_E and $\hat{F}$ in L_I (for 0-ary functions, F_c, $F_c = \Pi(\hat{F}_c)$), and (ΠP) sentences

$$\forall \hat{x}_1, \ldots, \hat{x}_n(\hat{D}(\hat{x}_1) \wedge \ldots \wedge \hat{D}(\hat{x}_n) \supset (P(\Pi(\hat{x}_1), \ldots, \Pi(\hat{x}_n)) \equiv \hat{P}(\hat{x}_1, \ldots, \hat{x}_n)))$$

for each n-ary predicate constant, P, in L_E and $\hat{P}$ in $\hat{\mathbf{P}}$ (for propositions, P_r, $P_r \equiv \hat{P}_r$),and ($\Pi =$) sentence

$$\forall \hat{x}_1, \hat{x}_2(\hat{D}(\hat{x}_1) \wedge \hat{D}(\hat{x}_2) \supset (\Pi(\hat{x}_1) = \Pi(\hat{x}_2) \equiv \hat{x}_1 \hat{=} \hat{x}_2)).$$

Π-axiom($\Pi; \hat{D}; \hat{=}$) states that the function, Π, can be considered as a homomorphic function of inner world into outer world which preserves the relations and functions.

If we assume that (d) and (e) hold, $A^{conn(\Pi;\hat{D})}$ can be transformed into another sentence, $\hat{A}^{conn(\hat{D})}$, in which no symbols in L_E occur. Next proposition shows that.

Definition. *Simple connotativization of A* is to replace each universal quantifier $\forall x B(x)$ in A by $\forall \hat{x}(\hat{D}(\hat{x}) \supset B(\hat{x}))$, each existential quantifier $\exists x B(x)$ by $\exists \hat{x}(\hat{D}(\hat{x}) \wedge B(\hat{x}))$, where $B(x)$ is a sentence in which x occurs free, and to replace F by $\hat{F}$ for each n-ary function, F, P by $\hat{P}$ for each n-ary predicate, P, and each equality $=$ by $\hat{=}$.

The result of simple connotativization is denoted by $\hat{A}^{conn(\hat{D})}$.

Note that $\hat{A}^{conn(\hat{D})}$ is written completely in $L_I \cup L_L$.

Proposition 1.

$$\hat{F}-axiom(\hat{D}) \wedge \Pi-axiom(\Pi;\hat{D};\hat{=}) \supset A^{conn(\Pi;\hat{D})} \equiv \hat{A}^{conn(\hat{D})}.$$

For this reason, we say that $\hat{F}$ ($\hat{P}, \hat{=}$) *is connotativization of* F ($P, =$), and also, for a tuple of predicates and / or functions, $\mathbf{K}$, we say that $\hat{\mathbf{K}}$ *is connotativization of* $\mathbf{K}$ if each predicate or function in $\hat{\mathbf{K}}$ is connotativization of its corresponding predicate or function in $\mathbf{K}$. In the remainder of this paper, we usually assume that $\hat{\mathbf{K}}$ denotes connotativization of $\mathbf{K}$.

CONNOTATIVE CIRCUMSCRIPTION

Now, we can introduce *connotative circumscription*. Connotative circumscription expresses minimization of the set, $\hat{C}$, of names, $\hat{x}$, denotating objects, $\Pi(\hat{x})$, that satisfy certain predicate C. That is, it is to minimize $\hat{C}$ in the conjunction of the sentences (a), (b), (c), (d) and (e).

$A_N \wedge A^{conn(\Pi;\hat{D})} \wedge \hat{F}-axiom(\hat{D}) \wedge \Pi-axiom(\Pi;\hat{D};\hat{=}) \wedge U_N(\hat{F})$ is denoted by $\hat{A}(\Pi;\hat{D};\hat{=};\hat{\mathbf{F}})$, or simply abbreviated to $\hat{A}(\hat{\mathbf{F}})$ if it is clear by the context.

$[\mathbf{K}/\mathbf{L}]$ is substitution of L for K, such that $\mathbf{K}$ and $\mathbf{L}$ are similar, that is, $A[\mathbf{K}/\mathbf{L}]$ represents a result of substitution of $\mathbf{L}$ for $\mathbf{K}$ in A, where A is a sentence. We abbreviate this to $[\mathbf{L}]$ if it is clear by the context.

$\mathbf{K} \leq \mathbf{L}$ means that $\mathbf{K}$ and $\mathbf{L}$ are similar, and stands for

$$\forall \mathbf{x}(K_1(\mathbf{x}) \supset L_1(\mathbf{x})) \wedge \ldots \wedge \forall \mathbf{x}(K_m(\mathbf{x}) \supset L_m(\mathbf{x})),$$

assuming that $\mathbf{K} = K_1, \ldots, K_m$ and $\mathbf{L} = L_1, \ldots, L_m$. And $\mathbf{K} < \mathbf{L}$ stands for $(\mathbf{K} \leq \mathbf{L}) \wedge \neg(\mathbf{L} \leq \mathbf{K})$.

Definition. Let $\mathbf{C}, \mathbf{Z}$ be a tuple of distinct predicate constants (which equality, $=$, is allowed to be a member of), and $\hat{\mathbf{C}}, \hat{\mathbf{Z}}$ be connotativization of $\mathbf{C}, \mathbf{Z}$. Also, let $\mathbf{F}$ be a tuple of distinct functions and $\hat{\mathbf{F}}$ be connotativization of $\mathbf{F}$. $\mathbf{E}$ is a tuple of all predicates and all functions in L_E. Let A be a closed sentence.

The connotative circumscription of $\mathbf{C}$ *with variable* $\mathbf{Z}$ *in* A *w.r.t.* $\mathbf{F}$ is the sentence

$$\hat{A}(\hat{\mathbf{F}}) \wedge \neg\exists \pi, \hat{\mathbf{c}}, \hat{\mathbf{z}}, \mathbf{e}.(\hat{\mathbf{A}}(\hat{\mathbf{F}})[\pi, \hat{\mathbf{c}}, \hat{\mathbf{z}}, \mathbf{e}] \wedge \hat{\mathbf{c}} < \hat{\mathbf{C}}),$$

denoted by $C-Cir(A;\mathbf{C};\mathbf{Z};\mathbf{F})$.

The connotative circumscription can be simplified remarkably by simple connotativization.

Proposition 2.

(1) $C-Cir(A;\mathbf{C};\mathbf{Z};\mathbf{F})$

$$\equiv \hat{A}(\hat{\mathbf{F}}) \wedge \neg\exists \pi, \hat{\mathbf{c}}, \hat{\mathbf{z}}((\hat{A}^{conn(\hat{D})} \wedge \hat{F}-axiom(\hat{D}) \wedge \hat{E}-axiom(\Pi;\hat{D};\hat{=}))[\pi, \hat{\mathbf{c}}, \hat{\mathbf{z}}] \wedge \hat{\mathbf{c}} < \hat{\mathbf{C}}),$$

where $\hat{E}-axiom(\Pi;\hat{D};\hat{=})$ is $(\Pi =)$ sentence, and,

(2) moreover, when if equality does not occur in $\mathbf{C}, \mathbf{Z}$,

$$C-Cir(A;\mathbf{C};\mathbf{Z};\mathbf{F}) \equiv \hat{A}(\hat{\mathbf{F}}) \wedge \neg\exists \hat{\mathbf{c}}, \hat{\mathbf{z}}((\hat{A}^{conn(\hat{D})} \wedge \hat{F}-axiom(\hat{D}))[\hat{\mathbf{c}}, \hat{\mathbf{z}}] \wedge \hat{\mathbf{c}} < \hat{\mathbf{C}}).$$

Note that $\hat{A}^{conn(\hat{D})} \wedge \hat{F}-axiom(\hat{D}) \wedge \hat{E}-axiom(\Pi;\hat{D};\hat{=})$ is a sentence of $L_I \cup L_L$.

In the reminder of this paper, we use the simpler definition of $C-Cir(A;\mathbf{C};\mathbf{Z};\mathbf{F})$ using proposition 2. So, $\hat{A}(\hat{\mathbf{F}})$ stands for

$$A_N \wedge A^{conn(\hat{D})} \wedge \hat{F}-axiom(\hat{D}) \wedge \Pi-axiom(\Pi;\hat{D};\hat{=}) \wedge U_N(\hat{\mathbf{F}})$$

and $C{-}Cir(A;\mathbf{C};\mathbf{Z};\mathbf{F})$ denotes

$$\hat{A}(\hat{\mathbf{F}}) \wedge \neg\exists\pi,\hat{\mathbf{c}},\hat{\mathbf{z}}((\hat{A}^{conn(\hat{D})} \wedge \hat{F}{-}axiom(\hat{D}) \wedge \hat{E}{-}axiom(\Pi;\hat{D};\dot{=}))[\pi,\hat{\mathbf{c}},\hat{\mathbf{z}}] \wedge \hat{\mathbf{c}} < \hat{\mathbf{C}}).$$

Example 1 (continued). We connotatively circumscribe Ab with variable Man. In this case, we can use the result of proposition 2. $C{-}Cir(A;Ab;Man;Anonym,Tweety)$ is

$$A_N \wedge \hat{A}^{conn(\hat{D})} \wedge \hat{F}{-}axiom(\hat{D}) \wedge \Pi{-}axiom(\Pi;\hat{D};\dot{=}) \wedge U_N(\widehat{Anonym},\widehat{Tweety})$$

$$\wedge\neg\exists\hat{ab},\widehat{man}((\hat{A}^{conn(\hat{D})} \wedge \hat{F}{-}axiom(\hat{D}))[\hat{ab},\widehat{man}] \wedge \hat{ab} < \widehat{Ab}),$$

where $\neg\exists\hat{ab},\widehat{man}((\hat{A}^{conn(\hat{D})} \wedge \hat{F}{-}axiom(\hat{D}))[\hat{ab},\widehat{man}] \wedge \hat{ab} < \widehat{Ab})$ is

$$\forall\hat{ab},\widehat{man}((\hat{A}^{conn(\hat{D})} \wedge \hat{F}{-}axiom(\hat{D}))[\hat{ab},\widehat{man}] \wedge \forall\hat{x}(\hat{ab}(\hat{x}) \supset \widehat{Ab}(\hat{x})) \supset \forall\hat{x}(\widehat{Ab}(\hat{x}) \supset \hat{ab}(\hat{x}))),$$

$\hat{A}^{conn(\hat{D})}$ is

$$\forall\hat{x}(\hat{D}(\hat{x}) \wedge \widehat{Knocks}(\hat{x}) \wedge \neg\widehat{Ab}(\hat{x}) \supset \widehat{Man}(\hat{x}))$$
$$\wedge\widehat{Knocks}(\widehat{Anonym})$$
$$\wedge\widehat{Ab}(\widehat{Tweety})$$
$$\wedge\neg\widehat{Man}(\widehat{Tweety}),$$

$\hat{F}{-}axiom(\hat{D})$ is

$$\hat{D}(\widehat{Anonym}) \wedge \hat{D}(\widehat{Tweety}),$$

$\Pi{-}axiom(\Pi;\hat{D};\dot{=})$ is

$$Anonym = \Pi(\widehat{Anonym})$$
$$\wedge Tweety = \Pi(\widehat{Tweety})$$
$$\wedge\forall\hat{x}(\hat{D}(\hat{x}) \supset (Ab(\Pi(\hat{x})) \equiv \widehat{Ab}(\hat{x})))$$
$$\wedge\forall\hat{x}(\hat{D}(\hat{x}) \supset (Man(\Pi(\hat{x})) \equiv \widehat{Man}(\hat{x})))$$
$$\wedge\forall\hat{x}(\hat{D}(\hat{x}) \supset (Knocks(\Pi(\hat{x})) \equiv \widehat{Knocks}(\hat{x})))$$
$$\wedge\forall\hat{x}_1,\hat{x}_2(\hat{D}(\hat{x}_1) \wedge \hat{D}(\hat{x}_2) \supset (\Pi(\hat{x}_1) = \Pi(\hat{x}_2) \equiv \hat{x}_1 \dot{=} \hat{x}_2)).$$

and $U_N(\widehat{Anonym},\widehat{Tweety})$ is

$$\widehat{Anonym} = \dot{A}nonym \wedge \widehat{Tweety} = \dot{T}weety.$$

In $C - Cir(A;Ab;Man)$, if we substitute $\hat{ab} = \lambda\hat{x}(\hat{x} = \widehat{Tweety})$ and $\widehat{man} = \lambda\hat{x}(T)$, where T stands for tautology (that is, $T \equiv (P_r \vee \neg P_r)$ for a proposition,P_r), then it yields

$$\forall\hat{x}(\widehat{Ab}(\hat{x}) \supset \hat{x} = \widehat{Tweety}),$$

that is,

$$\forall\hat{x}(\hat{D}(\hat{x}) \wedge Ab(\Pi(\hat{x})) \supset \hat{x} = \widehat{Tweety}).$$

This says that only the object named $\widehat{Tweety}$ satisfies Ab. As $\widehat{Anonym}$ and $\widehat{Tweety}$ is equivalent to natural numbers by the unique name axiom, for instance, $\widehat{Anonym} = 0$ and $\widehat{Tweety} = 1$, we obtain

$$\neg\widehat{Ab}(\widehat{Anonym}) \wedge \widehat{Man}(\widehat{Anonym}),$$

which is equivalent to

$$\neg Ab(Anonym) \wedge Man(Anonym),$$

since $\hat{D}(\widehat{Anonym})$ and $\Pi-axiom$.

Note that this result saying "Anonym is not abnormal and a man", is the intended result but can never be yielded by usual circumscription. Moreover, $\forall\hat{x}(\hat{D}(\hat{x}) \wedge Ab(\Pi(\hat{x})) \supset \hat{x} = \widehat{Tweety})$ says nothing about objects with no name, so abnormal things may still exist somewhere. This shows that connotative treatment is a method to solve the problems of absence of abnormal things and limitation on equality.

Example 2. The unique name assumption. Connotative treatment is also successful in formalizing the unique name assumption. We connotatively circumscribe equality, $=$.

Example 2.a. Clark's equality theory

$\hat{A}(\hat{\mathbf{F}})$ is finitely axiomatized if L_E are finite. Let A be a sentence in which equality does not occur positively. By the result in proposition 2. (2), assuming that $\mathbf{F}$ involves any functions in A, the connotative circumscription of equality, $C-Cir(A; =; \mathbf{F})$ is

$$\hat{A}(\hat{\mathbf{F}}) \wedge \forall\pi, eq((\hat{A}^{conn(\hat{D})} \wedge \hat{F}-axiom(\hat{D}) \wedge \hat{E}-axiom(\Pi, \hat{D}, \hat{=}))[\pi, \hat{=}/eq] \wedge \forall\hat{x},\hat{y}(eq(\hat{x},\hat{y}) \supset \hat{x}\hat{=}\hat{y})$$

$$\supset \forall\hat{x},\hat{y}(\hat{x}\hat{=}\hat{y} \supset eq(\hat{x},\hat{y}))),$$

which simplifies to

$$\hat{A}(\hat{\mathbf{F}}) \wedge \forall\pi, eq(\hat{E}-axiom(\pi, \hat{D}, eq) \wedge \forall\hat{x},\hat{y}(eq(\hat{x},\hat{y}) \supset \hat{x}\hat{=}\hat{y})$$

$$\supset \forall\hat{x},\hat{y}(\hat{x}\hat{=}\hat{y} \supset eq(\hat{x},\hat{y}))),$$

since $\hat{=}$ does not occur positively in $\hat{A}^{conn(\hat{D})} \wedge \hat{F}-axiom(\hat{D})$, which does not affect the process of minimizing $\hat{=}$. Substituting $=$ for eq and $\lambda x(x)$ for π, we obtain

$$\forall\hat{x},\hat{y}(\hat{x}\hat{=}\hat{y} \supset \hat{x} = \hat{y}).$$

Therefore,

$$\forall\hat{x},\hat{y}(\hat{D}(\hat{x}) \wedge \hat{D}(\hat{y}) \wedge \Pi(\hat{x}) = \Pi(\hat{y}) \supset \hat{x} = \hat{y}).$$

This says that distinct names denotes distinct objects, which means a narrow definition of the unique name assumption itself [see Clark's Equality Theory; [2]].

Note that although there are infinite names (natural numbers) this formalization is finitely axiomatizable if L_E is finite.

Example 2.b. With a domain closure axiom.

Let A be a sentence

$$\forall x(x = Jekyll \vee x = Hyde) \wedge Man(Stevenson).$$

Here, we want to minimize equality, that is, consider the connotative circumscription of equality. In $C-Cir(A; =; Jekyll, Hyde, Stevenson)$ (see Example 2. a.), $\hat{A}^{conn(\hat{D})}$ is

$$\forall\hat{x}(\hat{D}(\hat{x}) \supset \hat{x}\hat{=}\widehat{Jekyll} \vee \hat{x}\hat{=}\widehat{Hyde}) \wedge \widehat{Man}(\widehat{Stevenson}),$$

and $\hat{F}-axiom(\hat{D})$ is

$$\hat{D}(\widehat{Jekyll}) \wedge \hat{D}(\widehat{Hyde}) \wedge \hat{D}(\widehat{Stevenson}).$$

If we substitute $\lambda x, y(x = y \vee x = \widehat{Jekyll} \wedge y = \widehat{Stevenson} \vee y = \widehat{Jekyll} \wedge x = \widehat{Stevenson})$ for $\hat{=}$, and Π_1 for π, such that if $x = \widehat{Stevenson}$ then $\Pi_1(x) = \widehat{Jekyll}$ else $\Pi_1(x) = x$, we obtain

$$\widehat{Stevenson}\hat{=}\widehat{Jekyll}$$

$$\supset \forall\hat{x},\hat{y}(\hat{x}\hat{=}\hat{y} \supset (\hat{x} = \hat{y} \vee \hat{x} = \widehat{Jekyll} \wedge \hat{y} = \widehat{Stevenson} \vee \hat{y} = \widehat{Jekyll} \wedge \hat{x} = \widehat{Stevenson})).$$

A_N, U_N and the result of substitution of $\widehat{Jekyll}, \widehat{Hyde}$ for x, y in the above sentence yields

$$\widehat{Stevenson}\hat{=}\widehat{Jekyll} \supset \neg(\widehat{Jekyll}\hat{=}\widehat{Hyde})$$

follows. Similarly,

$$\widehat{Stevenson} \hat{=} \widehat{Hyde} \supset \neg(\widehat{Jekyll} \hat{=} \widehat{Hyde})$$

holds. The above two results and $\hat{A}^{conn(\hat{D})}$ yields

$$(\widehat{Stevenson} \hat{=} \widehat{Hyde} \vee \widehat{Stevenson} \hat{=} \widehat{Jekyll}) \wedge \neg(\widehat{Jekyll} \hat{=} \widehat{Hyde})$$

follows. By $\Pi-axiom$ and $\hat{F}rm-axiom$, we obtain

$$(Stevenson = Hyde \vee Stevenson = Jekyll) \wedge Jekyll \neq Hyde$$

This shows the result of minimization of equality.

3 Options

In some applications, we may intend to obtain stronger results than the results from the connotative treatment of circumscription mentioned above. In this section, we briefly consider possible and interesting extension of connotative circumscription for obtaining stronger results.

1) "Absence of abnormal thing" is a criticism for formalizing common-sense knowledge by minimizing abnormality denotatively, and connotative circumscription evades the criticism by minimizing connotatively. For instance, in the Example 1, connotative minimization of abnormality brings a weaker result, "Only the object named $\widehat{Tweety}$ is abnormal", than the result by the usual minimization, "Only $Tweety$ is abnormal". However, in some application of circumscription, we may want to obtain such a strong results. In the case, we add the following (ΠOP) sentence to $\Pi-axiom$;

$$\forall x_1, \ldots, x_n(C(x_1, \ldots, x_n) \supset \exists \hat{x_1}(\hat{D}(\hat{x_1}) \wedge x_1 = \Pi(\hat{x_1})) \wedge \ldots \wedge \exists \hat{x_n}(\hat{D}(\hat{x_n}) \wedge x_n = \Pi(\hat{x_n}))).$$

It says that each object of a tuple which satisfies C has his own name, which means that Π maps the extension of $\hat{C}$ onto the extension of C. In Example 1, let us assume that we add a sentence obtained by substitution of Ab for C in (ΠOP) sentence to $\Pi-axiom$. We still obtain

$$\forall \hat{x}(\hat{D}(\hat{x}) \wedge Ab(\Pi(\hat{x})) \supset \hat{x} = \widehat{Tweety}).$$

This result and (ΠOP) sentence yields

$$\forall x(Ab(x) \supset \exists \hat{x}(\hat{D}(\hat{x}) \wedge x = \Pi(\hat{x}) \wedge (\widehat{Ab}(\hat{x}) \supset \hat{x} = \widehat{Tweety}))),$$

which simplifies to

$$\forall x(Ab(x) \supset x = Tweety).$$

Note that we can still obtain $\neg Ab(Anonym) \wedge Man(Anonym)$.

2) Another interesting option is related to *Skolemization*. Consider the following example.

Example 1'. Let a sentence, A, be

$$\forall x(Knocks(x) \wedge \neg Ab(x) \supset Man(x)),$$
$$\wedge \exists x Knocks(x),$$
$$\wedge Ab(Tweety),$$
$$\wedge \neg Man(Tweety).$$

Same to Example 1, "someone" knocks on the door, though we have no name of it. Should we guess that it would be a man? I can not tell which is better. It would depend on the situation. Anyway, if we think that we should guess so, we can obtain the intended results by means of Skolemization in the inner world.

We skolemize in connotativization of quantifiers. The result is

$$\forall \hat{x}(\hat{D}(\hat{x}) \wedge \widehat{Knocks}(\hat{x}) \wedge \neg \widehat{Ab}(\hat{x}) \supset \widehat{Man}(\hat{x}))$$

$$\wedge \widehat{Knocks}(\hat{G})$$

$$\wedge \widehat{Ab}(\widehat{Tweety})$$

$$\wedge \neg \widehat{Man}(\widehat{Tweety}),$$

where $\hat{G}$ is a Skolem function. The connotative circumscription of Ab with variable Man still yields

$$\widehat{Knocks}(\hat{G}) \wedge \neg \widehat{Ab}(\hat{G}) \wedge \widehat{Man}(\hat{G}),$$

which is equivalent to

$$Knocks(\Pi(\hat{G})) \wedge \neg Ab(\Pi(\hat{G})) \wedge Man(\Pi(\hat{G})).$$

This implies

$$\exists x(Knocks(x) \wedge \neg Ab(x) \wedge Man(x)).$$

Connotative circumscription based on Skolemized connotativization (we say, *Skolemized connotative circumscription*) has an important aspect. Skolemized connotativization yields a universal sentence (that is, $\forall \mathbf{x} A$, where $\mathbf{x}$ is a tuple of object variables, and A is quantifier-free) therefore, the next proposition holds.

Proposition 3. Let A be a consistent closed sentence, and $\mathbf{F}$ involve any functions in A. Any Skolemized connotative circumscription w.r.t. $\mathbf{F}$ in A is consistent.

Note that circumscription is inconsistent generally [3].

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