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Heuristics and more Heuristics: Toward Solving Harder Quasigroup Problems

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Heuristics and more Heuristics: Toward Solving Harder Quasigroup Problems* (Extended Abstract)

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1 INTRODUCTION

In 1992 ICOT's MGTP(model generation theorem prover)[4] , [3] made an obvious breakthrough in deciding some finite quasigroup existence problems by a method of model enumeration, after some tries and a success in China[7] and Australia[5].

Since then, two types of systems, such Mark Stickle's DDPP, an implementation of the Davis-Putnam algorithm, and John Slaney's Finder, a finite-domain constraint based enumeration system, have achieved solutions to harder problems in the domain and contributed to great progress in design theory in discrete mathematics[6]. In this process, the major reasons for new breakthroughs are strongly related to "how to minimize guessing," or how to cleverly choose the next place to assign a value. Choosing a cell with the least alternative values in the latin square or choosing a literal in one of the least length positive clauses is the common criterion of all systems. And heuristics we found have taken the role of shortening the least length positive clauses. A list of the heuristics and the reasons why they are effective will clarify the advantage of representing the problem as propositions. One reason is that all heuristics are represented declaratively and they are independent from the program and are very easy to verify and modify.

But there is no reason to say that all possible heuristics can be represented as propositions. 'Look ahead' is a very costly but useful method to see if a search for more heuristics is promising or not. Although the result willll be carefully investigated, this result of the experiment is somewhat attractive.

2 HEURISTICS

2.1 Initial Clause Set

For a finite (idempotent-)quasigroup $< B, * >$[1] the following is one of the minimal set of clauses that MGTP used for getting some new results. We can safely say that $B = \{1, \ldots, n\}$.

1. **domain**
   for all a, b in B $a * b = 1 \vee \ldots \vee a * b = n$

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2. **quasigroup**
   for all different a, b in B $\neg X * a = Y \vee \neg X * b = Y$
   for all different a, b in B $\neg a * X = Y \vee \neg b * X = Y$

3. **idempotence**
   for all a in B $a * a = a$

4. **identities**
   QG5:$(yx.y)y = y$
   for all a, b, u, v, x in B if $a \neq x$ then $\neg b * a = u \vee \neg u * b = v \vee \neg v * b = x$
   or equivalently $\neg b * a = u \vee \neg u * b = v \vee v * b = a$

2.2 Heuristics

1. **Isomorphic Search Space**

   $x \circ v \geq x - 1$ was a suboptimal heuristic to avoid searching isomorphic models. A slightly better method was obtained by using a kind of normal form of isomorphic set of models. Over the equational heuristic, to compell the cycles in the $x \circ 1$ column occur in monotone decreasing order of length sometimes bring a few times speed-up, but still less than complete. The symmetry of quasigroups give you three choices of position of a row, a column, or a value and the effect is problem dependent.

   Finding a useful single canonical form of isomorphic set of quasigroups is still open.

2. **Symmetry of Quasigroups**

   As Mendelshon remarked[1], quasigroups are three mutually orthogonal n^2 arrays and two extra equations((2),(3)) from the original(1) derived by using this property worked as effective extra heuristics.

   Example 1 *:QG5*
   $(yx.y)y = y$ *(1)*
   $(y.xy)y = x$ *(2)*
   $y(xy.y) = x$ *(3)*

   And a clausal form expression of each of these identities respectively is
   $p(Y,X,U) \vee -p(U,Y,V) \vee p(V,Y,X)$ (4)
   $-p(Y,X,U) \vee p(U,Y,V) \vee -p(V,Y,X)$ (5)
   $p(Y,X,U) \vee -p(U,Y,V) \vee -p(V,Y,X)$ (6)

   (5) and (6) reinforce (4) to produce much more units and shrink the search space.

3. **Surjection**
   The heuristic 'surjection' (If $\neg p(a,\_,b)$ for all _ except for a single c then $p(a,b,c)$) in Finder was the origin of extra positive clauses.
   $p(a,1,b) \vee \ldots \vee p(a,n,b)$
   $p(1,a,b) \vee \ldots \vee p(n,a,b)$
   These positive clauses can be the candidate of case split and a natural extension of surjection.

Note that all the extra heuristics could be obtained by introducing new clauses.

Problem		MGTP(original)	Davis Putnam	Look Ahead
QG3	7	183	24	5
	8	3,875	31	5
QG4	7	123	33	5
	8	3,516	501	44
QG5	7	9	2	0
	8	34	8	0
	9	239	15	1
	10	7,026	38	5
	12	2,749,676	959	83
QG6	7	7	2	1
	8	20	6	0
	9	160	12	0
	10	2,881	52	8
QG7	7	182	16	1
	8	160	32	2
	9	37,026	190	17

Table 1: Failed branches

3 MORE HEURISTICS

The more heuristics enforce unit propagation, the smaller the search space becomes. So searching heuristics entered even deeply in the domain of quasigroup theory. Some achievements are obtained by taking part in the mathematician's research.

1. generate and test by incomplete construction algorithms
   Mathematicians are using this method. (It is similar to a construction method of magic squares.) H. Zhang of Ohio State University implemented Bennett's method and obtained some new results.

2. check if a solution of one problem satisficed other problems' constraints
   Stickel found that a conjugate of the solution of QG4.12 is the solution of QG2.12.

But these are unrelated to our original approach and no help to shrink the search space of the enumeration method.

3.1 Look Ahead

We tested a new technique, 'look ahead,' to see if there are more effective heuristics to reduce the search space. Look ahead assigns a truh value of a proposition iff contradiction can be deducible by hypothesing opposite value and applying only unit propagation. For example, the value of true or false of $a * b = c$ is not decided in the model candidate and if the true assignment is refutable by unit propagation then $\neg a * b = c$. Look ahead is easy to attach to the solvers. Although 'Look ahead' is a heavy procedure, the result of the experiment in Table 1 encourages a search for further heuristics.

3.2 Look Ahead without Look Ahead

Look ahead seems to be too heavy to solve harder problems. However, the number of unassigned atoms was large(initially 1000 for order 10 problem) and the unit propagation would be heavily duplicated. The typical example of this redundancy is that if a is deducible by asserting b then look ahead of asserting a is useless. The second reason is that the number of new 2-clauses are relatively much less than the clauses in the closure but look ahead does not care what unit propagation is new or not.

We are seeking a light algorithm to avoid most of the redundant computation in the 'look ahead'. For the Davis-Putnam procedure, a possible extension is the 2-closure[2] algorithm. Roughly speaking, the 2-closure algorithm is building and maintaining a deductive closure of all length 2 clauses along with the D.P. procedure. This is polynomial order and can avoid redundant computation by propagating only the changes.

4 FUTURE WORK

The 2-closure algorithm has not yet turned out to be useful for quasigroup problems because of a lack of experiments with reasonably large size problems. We first need to make such an experiment.

Another possible test is to build a selection function of a literal from the least positive clause. Useful information such as the number of immediately deducible units by asserting an atom becomes cheaper weighting function of literals.

References

[1] F. Bennett, *Quasigroup Identities and Mendelshon Designs*, **Canadian Journal of Mathematics** 41 (1989), pp. 341–368.

[2] A. Gelder and Y. Tsuji, *Satisfiability Testing with More Reasoning and Less Guessing* Preprint, 1994.

[3] H. Fujita and R. Hasegawa, *A Model Generation Theorem Prover in KL1 Using Ramified-Stack Algorithm*, **Proc. of ICLP91**, pp.535-548, 1991.

[4] M. Fujita, J. Slaney & F. Bennett, *Automatic Generation of Some Results in Finite Algebra*, **Proc. International Joint Conference on Artificial Intelligence**, 1993.

[5] J. Slaney, *FINDER, Finite Domain Enumerator: Version 2.0 Notes and Guide*, technical report TR-ARP-1/92, Automated Reasoning Project, Australian National University, 1992.

[6] J., Slaney, M. Fujita, & M. Stickel. *Automated reasoning and exhaustive search: Quasigroup existence problems*, To appear in **Computers and Mathematics with Applications**, 1993.

[7] J. Zhang, *Search for Idempotent Models of Quasigroup Identities*, Typescript, Institute of Software, Academia Sinica, Beijing.