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New Results in Mathematics by a Parallel Theorem Prover on the Parallel Inference Machine (Abstract)

並列推論マシン上の並列定理証明系による有限代数の新事実 ー概要ー

by

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Abstract

This abstract reports results in mathematics obtained by a parallel theorem prover developed at ICOT. MGTP/G[FH92] is a family of tableau or model generation theorem provers developed at ICOT on range-restricted[1] first order problems based on SATCHMO[MB88].

For MGTP/G, case splitting is the critical source of both the efficiency and the combinatorial explosion. MGTP/G/MERC/PF, which is one of our solution to this latter problem, uses the following two simple but effective techniques. One is to simplify a disjunctive model element by falsifying each literal independently by a hyperresolution step with negative clauses and model elements. By this method each disjunctive clause may shrink before used for case splitting. The other is to order clauses by the number of literals. These dynamic controls of model generation prevent the model generator from combinatorially exploding by doing useless case splitting. Thanks to these mechanisms, MGTP/G/MERC/PF has solved some open problems in finite algebra(Table1).

[1] A clause is range-restricted iff every variable in the consequent occurs the antecedent at least once.

Problem	Failed Branches by MERC/PF	Failed Branches by MERC	No. of Solutions
10 Queens	4,942	312,612	724
11 Queens	21,528	1,639,781	2,680
Bennett 4	1	104	0
Bennett 5	1	2,400	1
Bennett 6	3	179,171	0
Bennett 7	6	52,249,612	3
Bennett 8	33	-	1
*Bennett 9	239	-	0
*Bennett 10	7,026	-	0
Bennett 11	51,899	-	5
*Bennett 12	2,749,676	-	0

Table 1: The Effect of Partial Falsification

In [Be89], Bennett showed the existence of idempotent quasigroup[2] such that every pair of elements satisfies the equation $((y * x) * y) * y = x$ of order $n \geq 1$ and $n \neq 2, 3, 4, 6$. He has left open 56 orders $n \in E$ [3]. When Q is finite, you can create the multiplication table. Table 2 shows a table for order 5 Bennett's quasigroup. Orthogonal result is identical to each row and column. From $3 * 4 = 1$, $1 * 3 = 2$ and $2 * 3 = 4$, $3 * 4 * 3 * 3$ becomes 4 .

MGTP/G/MERC/PF on the Parallel Inference Machine(PIMM-256) at ICOT showed that no such quasigroup of order 9, 10, 12 exists. PIMM is an MIMD inference machine with 256 processors of 1M LIPS(Logical Instruction Per Second) each. Problem of order 12 needed about 13715 cpu sec(3 hours 49 min) on PIMM-256 for 2749676 case splitting branches. Although Finder[Sl92] had solved order 9 in 26917 seconds on a SPARCserver 670,

[2] A *quasigroup* is an ordered pair $(Q, *)$, where Q is a set and $*$ is a binary operation on Q such that the equations $a * x = b$ and $y * a = b$ are uniquely solvable for every pair of elements a, b in Q. A quasigroup$(Q, *)$ is called *idempotent* iff for every element a of Q satisfies the equation $a * a = a$.

[3] E={9, 10, 12, 13, 14, 15, 16, 18, 20, 22, 24, 26, 28, 30, 34, 38, 39, 42, 44, 46, 51, 52, 58, 60, 62, 66, 68, 70, 72, 74, 75, 76, 86, 87, 90, 94, 96, 98, 99, 100, 102, 106, 108, 110, 114, 116, 118, 122, 132, 142, 146, 154, 158, 164, 170, 174}

	1	2	3	4	5
1	1	3	2	5	4
2	5	2	4	3	1
3	4	5	3	1	2
4	2	1	5	4	3
5	3	4	1	2	5

Table 2: Order 5 Bennett Quasigroup

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dom(1),dom(2),dom(3),dom(4),dom(5),dom(6):- true.
p(M,N,1);p(M,N,2);p(M,N,3);p(M,N,4);p(M,N,5);p(M,N,6):- dom(M),dom(N).
false:- p(X,6,Y),{Y+1<X}.
false:- p(X,X,U),{X≠U}.
false:- p(X,Y,U),p(X,Y1,U),{Y≠Y1}.
false:- p(X,Y,U),p(X1,Y,U),{X≠X1}.
false:- p(E,X,Y),p(Y,E,Z),p(Z,E,U),{X≠U}.
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Figure 1: Order 6 Bennett Problem for MGTP

order 10 and 12 are new results. No existence of order 10 non idempotent quasigroup with above equation was another solved open problem. This took 13101 cpu-sec(3h 39min) for 4473508 case splitting branches.

The problem description for MGTP theorem prover is very simple(Fig.1).

We could get new results with such naive representation. This gives us a hope to find mathematical properties for pruning search space and trying larger orders. Table3 shows the execution time by MGTP/G/MERC/PF. Remark that Bennett 9 was measured on single-processor.

References

[Be89] Bennett, F. E., "QUASIGROUP IDENTITIES AND MENDEL-

Problem	Time(Sec)
11Queens	2.3
12Queens	7.0
13Queens	27.9
14Queens	128.0
*Bennett 9	2116
*Bennett 10	66
Bennett 11	236
*Bennett 12	13,715
No Id 8	42
No Id 9	521
*No Id 10	13,101

Table 3: Execution Time of MGTP/G/MERC/PF on PIM/m-256

SOHN DESIGNS," in *Can. J. Math., Vol. XLI, No. 2, pp. 341-368*, 1989.

[FH92] Fujita, M., and Hasegawa, R., Koshimura, M., Fujita, H., "Model Generation Theorem Provers on A Parallel Inference Machine," in *Proc. of FGCS'92*, to appear, 1992.

[MB88] Manthey, R. and Bry, F.,"SATCHMO: a theorem prover implemented in Prolog, " in *Proc. of CADE 88, Argonne, illinois*, 1988.

[Sl92] Slaney, J. K., "FINDER Finite Domain Enumerator VERSION2.0 NOTES AND GUIDE," from the public domain softwares, The Australian National University, 1992.