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Designing the Optimal Structure of a Multi-Axis Gearbox

by

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Abstract.

A gearbox is one of important parts of a machine tool like a lathe. We propose a new designing algorithm for a gearbox, regarding its design problem as an optimization problem under constraints. In this problem, the structure of a gearbox is optimized by making it as narrow as possible, and constraints are: no interference between gears and no simultaneous meshing of different gears.

By conventional method, a designer must design a gearbox depending only on his experience and knowledge about a few simple structure examples like a two-axis-two-step gearbox. And a gearbox designed in such a way is not guaranteed to be optimal. On the other hand, by our algorithm, one can systematically design a gearbox which has an optimal structure. In other words, our proposal changes the designing from a conventional method based on designer's experience and knowledge into a systematical method guaranteed to lead to optimal solutions.

In this paper, we show that this problem results in a combinatorial optimization problem in finite domain. And at the end of this paper, we show some design examples and results of implementation of this algorithm.

1 Introduction

In this paper, we study the designing problem of a multi-axis gearbox. A gearbox is one of important parts of a machine tool and it is necessary to design it as small as possible because a larger structure has lower stiffness, lower strength, and needs higher cost than a smaller one.

But conventionally, the designing method of a gearbox is not systematically formulated and a designer must design it depending only on his experience and knowledge about a few simple structure examples like a two-axis-two-step gearbox. And a gearbox designed in such a way is not guaranteed to be optimal.

Now we propose a new designing algorithm for a gearbox, regarding its design problem as an optimization problem under constraints. In this problem, the structure of a gearbox is optimized by making it as narrow as possible, and constraints are: no interference between gears and no simultaneous meshing of different gears. In other words, we determine the positions of gears to minimize the overall width of a gearbox under these constraints. By our formulation this problem results in a combinatorial optimization problem in finite domain, though this problem is an optimization problem in infinite domain.

We start by showing an example of a gearbox and defining the design problem in section 2. In section 3, we describe the strategy for solving this problem. Because a multi-axis gearbox is constructed with partial structures of two- or three-axis gearboxes, this strategy has two steps: the first step is to design partial structures, and the second step is to construct the whole structure with these partial structures. In section 4, we describe the design algorithm for a partial structure, and in section 5 we explain the way to construct the whole structure with partial structures and that this problem results in a combinatorial optimization problem in finite domain. In section 6, we show some examples and results of implementation of our algorithm. Section 7 is the conclusion.

2 The Design Problem

2.1 A Gearbox

Figure 1 is an example of a gearbox[4]. The meshing gears are marked with an uppercase and a lowercase of the same letter. The first and third axes are fixed-gear axes and the second axis is a sliding-gear axis. On the sliding-gear axis, there are two sliding-gear units, A-B and C-D, which can slide along the axis independently of each other. Between the first and second axis there occur two gear change steps, that is, A-a and B-b, and between the second and third axis there also occur two gear change steps, C-c and D-d. Therefore, the total number of gear change steps of this gearbox is four because $2 \times 2 = 4$.

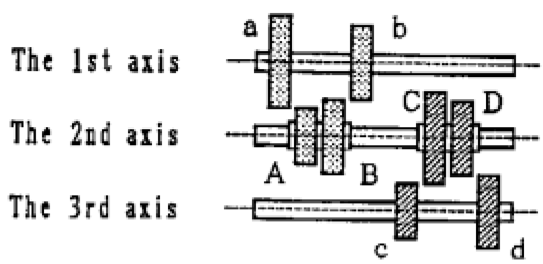

Figure 1: A gearbox

2.2 Definition of the Problem

The design problem of a multi-axis gearbox is defined as follows.

To obtain the positions of gears to minimize the overall width of a gearbox

The input are: 1) number of axes, 2) intervals between axes, 3) number of gear change steps at each interval, 4) partial speed ratio at each meshing, and 5) order of fixed-gear axes and sliding-gear axes.

To simplify this problem, it is assumed[3]:

(1) each gear has a unit width;

(2) two structures which are mirror images of each other are regarded as the same structure;

(3) fixed-gear axes and sliding-gear axes are arranged in turn.

The third assumption seems to lose generality of this problem, but a gearbox is actually designed like that in order to decrease sliding parts.

3 The Strategy

3.1 Two Steps of the Strategy

Because a multi-axis gearbox is constructed with several partial structures of two- or three-axis gearboxes, this strategy has two steps: the first step is to design partial structures, and the second step is to construct the whole structure with these partial structures. We will study the first step in section 4 and the second step in section 5, and in next subsection we explain that the whole structure is divided into several partial structures.

3.2 Dividing the Whole Structure

As described in 2.1, sliding-gear units slide independently of each other, in other words, these sliding regions have no common area. Therefore the whole structure can be divided into several partial structures by the border of sliding regions.

Figure 2 illustrates division of the whole structure. This is a five-axis gearbox, and the odd number axes are fixed-gear axes and the even number axes are sliding-gear axes. This gearbox has 36 gear change steps, three between the first and second axis, two between the second and third, two between the third and fourth, and three between the fourth and the last axis. Dotted lines are border between partial structures, and the whole structure is divided into three partial structures: a two-axis gearbox of fixed-sliding, a three-axis gearbox of sliding-fixed-sliding, and a two-axis gearbox of sliding-fixed.

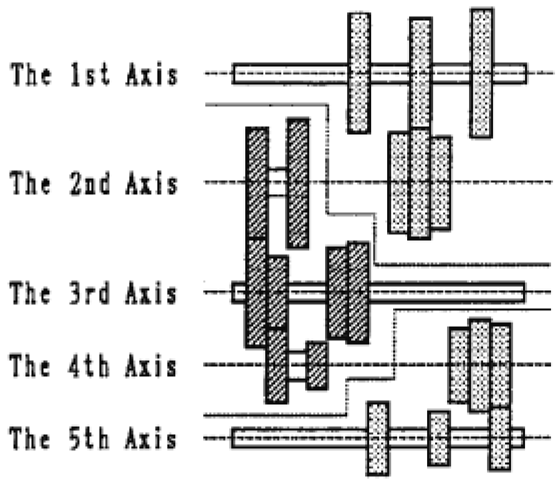

Figure 2: Division of the whole structure

This example shows that a multi-axis gearbox is constructed with two- or three-axis partial gearboxes.

4 Designing a Partial Structure

4.1 A Three-axis Gearbox

In this section, we consider only a three-axis gearbox of sliding-fixed-sliding as a partial structure, because a two-axis gearbox is regarded as the special case of a three-axis gearbox. Now we will design a three-axis gearbox in two steps[3]: the first step is to enumerate gear-ordering vectors and the second step is to determine the positions of gears for each gear-ordering vector.

4.2 A Gear-ordering Vector

When we determine the order of gears, we have to consider only the order of fixed gears[1]. A gear-ordering vector is described by a vector $\{a_i\}$. The suffix i stands for the order of the fixed gear, and the absolute value $|a_i|$ is the radius of it. When $a_i < 0$, the fixed gear meshes with a gear on the first axis, and when $a_i > 0$, it meshes with a gear on the third axis.

The gear-ordering vector of a structure in Figure 3 (a) is expressed as follows.

$$\{a_i\} = \{-3, 2, -5, 1, 4\}$$

In the case of Figure 3 (b) (called composite gear drive), the common gear is stood for by a list of two elements and the gear-ordering vector is expressed as follows.

$$\{a_i\} = \{-3, -2, [-1, 1], 4\}$$

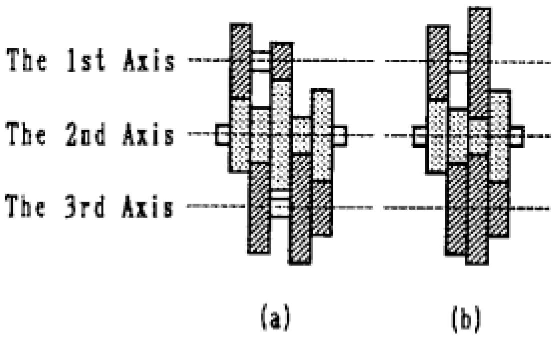

Figure 3: The order of gears

The size of the gear-ordering vector N is $(N^- + N^+ - N^C)$ and the total number of gear change steps is $(N^- \times N^+)$, where:

N^- is the number of gear change steps between the first and the second axis;
N^+ is the number of gear change steps between the second and the third axis;
N^C is the number of common gears.

4.3 The Positions of Gears

The positions of gears on the first, the second, and the third axis are expressed by vectors $\{s_i^-\}$, $\{f_i\}$, and $\{s_i^+\}$, whose sizes are the same as the size of the gear-ordering vector. The suffix i stands for the meshing gears, and the value of the element is the position of the gear. For example, the positions of gears of a gearbox in Figure 4 are expressed as follows.

$$\{a_i\} = \{-3, 2, -5, 1, 4\}$$
$$\{s_i^-\} = \{0, -, 4, -, -\}$$
$$\{f_i\} = \{0, 3, 6, 7, 12\}$$
$$\{s_i^+\} = \{-, 1, -, 7, 8\}$$

Now, we introduce gear-sliding vectors $\{d_i^-\}$ and $\{d_i^+\}$. They express the sliding distances necessary to mesh gears, and each element is defined as follows.

$$d_i^{\pm} = f_i - s_i^{\pm} \qquad (1)$$

In the case of Figure 4, they are:

$$\{d_i^-\} = \{0, -, 2, -, -\};$$
$$\{d_i^+\} = \{-, 2, -, 0, 4\}.$$

It is assumed that the minimum of elements of a gear-sliding vector is 0, and generality is not lost by this assumption. This gear-sliding vector play important parts of designing the optimal structure of a three-axis gearbox.

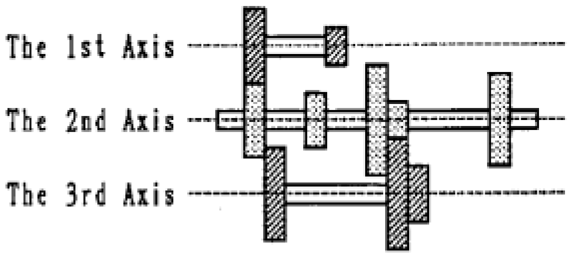

Figure 4: The positions of gears

4.4 The Constraints

4.4.1 No Interference between Gears

A fixed gear must not interfere with a sliding gear (Figure 5) and there is a possibility of interference between the ith sliding gear and the jth fixed gear when $|a_j| > |a_i|$. Therefore the constraint of no interference between gears is expressed[2]:
when $|a_j| > |a_i|$,

$$s_i^{\pm} + h_i + max(d_k^{\pm}) \leq f_j$$
$$\text{or } s_i^{\pm} \geq f_j + h_j;$$

where h_i is the width of the ith gear.

To eliminate $s_i^{\pm}$ by eq(1), it is rewritten:
when $|a_j| > |a_i|$,

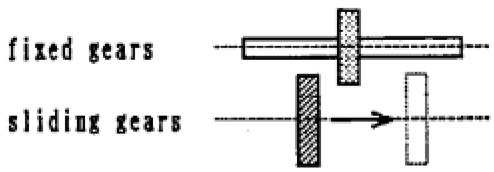

Figure 5: Interference between gears

$f_j - f_i \geq max(d_k^-) - d_i^- + h_i \ (a_i < 0);$
$f_j - f_i \geq max(d_k^+) - d_i^+ + h_i \ (a_i > 0);$

when $|a_j| < |a_i|$,

$f_j - fi \geq d_j^- + h_i \ (a_i < 0);$
$f_j - fi \geq d_j^+ + h_i \ (a_i > 0);$

where $j > i$.

4.4.2 No Simultaneous Meshing of Different gears

When you try to mesh gears by sliding the sliding-gear unit, other gears must be apart before you mesh the gears. Therefore the situation like Figure 6 is not allowed. This constraint is expressed[2]:

$d_i^{\pm} + h_i \leq d_j^{\pm} - h_j$ or $d_i^{\pm} - h_i \geq d_j^{\pm} + h_j;$

where $i \neq j$.

By these inequalities, we can write constraint for each $d_i^{\pm}$ when sequence of $d_i^{\pm}$ are given. For example, if sequence $0 = d_2^- < d_0^- < d_1^-$ are given, then the constraint is:

$d_2^- = 0,$
$d_0^- \geq d_2^- + h_2 + h_0,$
$d_1^- \geq d_0^- + h_0 + h_1.$

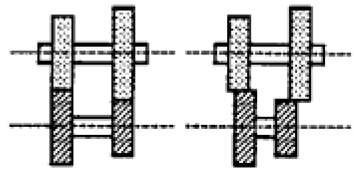

Figure 6: Simultaneous meshing

4.5 The Overall Width of a Three-axis Gearbox

The overall width of a three-axis gearbox W is determined by the sliding area of sliding gears on the first and the third axis[1], and W is calculated as follows.

$$W = max(s_i^{\pm} + h_i + max(d_k^{\pm})) - min(s_i^{\pm})$$
$$max(f_i - d_i^{\pm} + h_i + max(d_k^{\pm})) - min(f_i - d_i^{\pm}) \quad (2)$$

4.6 The Designing Procedure

In order to make a gearbox as narrow as possible, $d_i^{\pm}$ and f_i should be as small as possible. From the result of 4.4.2 and the assumption that $h_i = 1$, we conclude:

$\forall d_i^{\pm} \in \{0, 2, 4, \cdots, 2(N^{\pm} - 1)\}.$

Now the designing procedure of a three-axis gearbox is derived.

(1) Gear-ordering vectors $\{a_i\}$ are enumerated. When mirror images are excluded, constraint of $|a_0| < |a_{N-1}|$ is added.
(2) Gear-sliding vectors $\{d_i^{\pm}\}$ are enumerated.
(3) Sets of these three vectors, $\{a_i\}$ and $\{d_i^{\pm}\}$, are enumerated and the position vector of fixed gears $\{f_i\}$ is determined for each set, to choose the minimum value satisfying the constraint in 4.4.1 for f_i. It is assumed that $f_0 = 0$ and this assumption does not lose generality.
(4) The overall width W is calculated.
(5) Solutions to minimize the overall width W are selected.

We note that this designing problem results in a combinatorial optimization problem in finite domain.

4.7 Design Example

Figure 7 is a design example of a three-axis and six-gear-change-step gearbox, whose gear-ordering vector's elements are 1, 2, -3, 4 and -5. The overall width of this structure is 9.

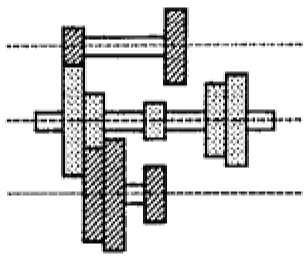

Figure 7: Design example

5 Designing the Whole Structure

5.1 Designing Procedure

The whole structure is designed as follows.

(1) Radii of gears are calculated from the input data. When there are two fixed gears which have same radii on the same axis, one common gear is set on the axis and composite gear drive is made.
(2) Partial structures are designed as described in section 4.
(3) The whole structures are constructed with partial structures, and the structures minimizing their overall width are selected.

The whole structure is constructed by setting sliding-gear units of different partial structures on the same axis. Therefore to construct the whole structure is to determine the distance between partial structures to be combined. When two partial structures to be combined are named A and B, the constraint is no interference[2]:

between fixed gears of A and sliding gears of B;
between sliding gears of A and sliding gears of B;
between sliding gears of A and fixed gears of B.

This problem is equal to the problem to determine the value of $D = f_A - f_B$, where f_A and f_B are the 0th fixed gear's positions of A and B. In 5.2, 5.3, and 5.4 each constraint is explained by using Figure 8.

5.2 Fixed Gears of A and Sliding Gears of B

A fixed gear of A must not interfere with a sliding gear of B. We can see whether a fixed gear interferes with a sliding gear by comparing the sum of their radii with the interval between the axes.

In Figure 8, the 0th sliding gear of B can be interfered with by the 0th, 2nd, and 3rd fixed gears of A. Because the sliding region of the sliding gear is $[f_B, f_B+3]$, the range of D is:

$$D \leq -3, D = 1, 8 \leq D.$$

And because of the constraint about the 1st sliding gear of B, the range of D is:

$$D \leq 0, 7 \leq D.$$

From these two results, the range of D is:

$$D \leq -3, 8 \leq D.$$

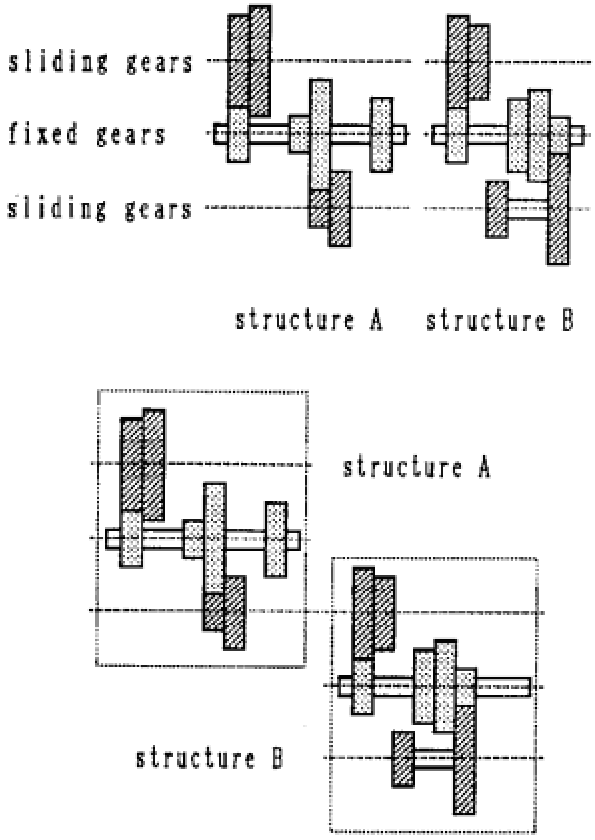

Figure 8: Construction of a whole structure

5.3 Sliding Gears of A and Sliding Gears of B

Two sliding region of different sliding-gear units have no common area. In Figure 8, because the sliding region of A is $[f_A + 4, f_A + 8]$ and the sliding region of B is $[f_B, f_B + 4]$, the range of D is:

$$D \leq 0, 8 \leq D.$$

5.4 Sliding Gears of A and Fixed Gears of B

In Figure 8, the 2nd sliding gear of A cannot be interfered with by any fixed gear of B, and the 3rd sliding gear can be done by the 1st and the 2nd fixed gear of B. Because its sliding region is $[f_A + 5, f_A + 8]$, the range of D is:

$$D \leq 0, 5 \leq D.$$

5.5 Construction of the Whole Structure

From the results of 5.2, 5.3 and 5.4 the range of D is:

$$D \leq -3,8 \leq D.$$

The overall width of the whole structure is minimized when $D = -3$. That structure is illustrated in Figure 9.

By this example, we found that the problem constructing the whole structure is also a combinatorial optimization problem. Therefore the problem of designing the optimal structure of a multi-axis gearbox results in a combinatorial optimization problem in finite domain.

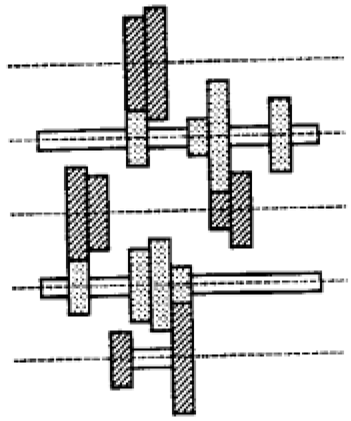

Figure 9: The whole structure

6 Results of Implementation

We have implemented this algorithm on the parallel machine, named Multi-PSI, in ICOT. Parallel processing is useful for solving this problem because it is solved quickly by dividing the search space. We made an experiment on the problem of designing a five-axis gearbox. The intervals between axes are all 1.0, the speed ratio series is a geometric series with a common ratio of 1.06, and the gear change steps are as follows.

a: 3-2-2-2 gear change steps
b: 3-2-2-3 gear change steps

Table 1 and Figure 10 show results of measurement, and in Figure 11, design examples are showed. The speedup is quite good, because this problem is a simple combinatorial optimization problem and communications between processors are few.

PEs	1	2	4	8	12
Run Time (s)					
a	104.9	54.3	27.5	14.2	10.2
b	1363.0	691.1	347.9	179.6	116.3
Speedup Ratio (to Sequential Program)					
a	1.0	1.9	3.7	7.2	10.1
b	1.0	1.9	3.8	7.4	11.4
Speedup Ratio to Number of PEs (%)					
a	100	94.5	93.3	90.3	83.8
b	100	96.1	95.4	92.4	95.2

Table 1: Results of measurement

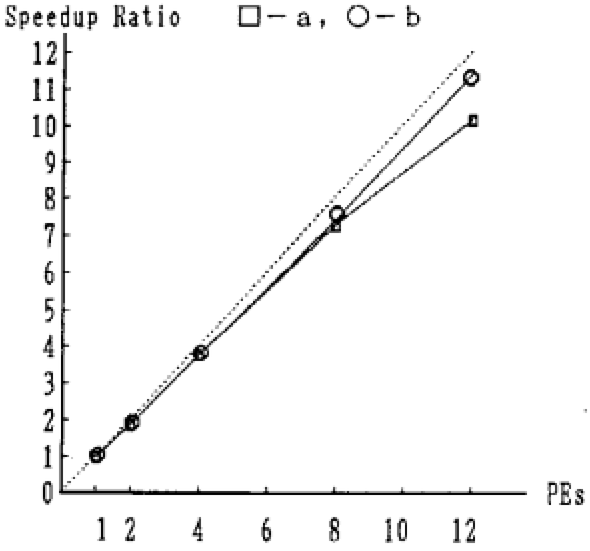

Figure 10: Speedup Ratio

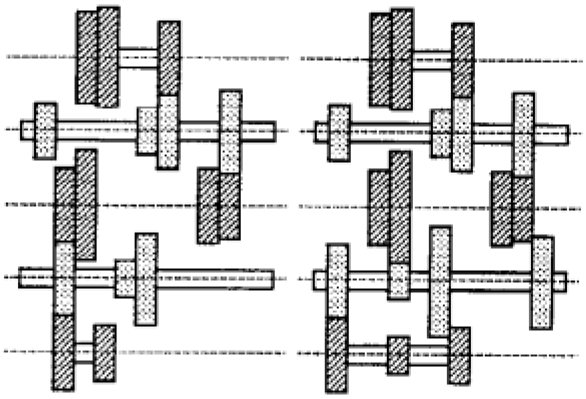

Figure 11: Design examples

7 Conclusion

We have proposed a new designing algorithm for a multi-axis gearbox. By our algorithm, a designer can design a gearbox systematically. In this paper we assumed that the width of all gears are 1, but without this assumption this algorithm can be applied easily (see 4.4.1 and 4.4.2).

Acknowledgement

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