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Logical Structure of Analogy
(PRELIMINARY REPORT)

by

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Abstract: This paper treats a general type of analogy which is described as follows: when two objects, B (called the *base*) and T (called the *target*), share a property S (called the *similarity*), it is conjectured that T satisfies another property P (called the *projected property*) which B satisfies as well. This type of analogy is analyzed formally, it is clarified that analogical reasoning is possible only if a certain form of rule, called the *analogical prime rule*, is a deductive theorem of a given theory. Also, this paper shows that, from the rule, together with two particular conjectures, an analogical conclusion is derived. A candidate is shown for a non-deductive inference system which can yield both conjectures.

1 Introduction

When we explain a process of reasoning by analogy, we may say that "An object T is similar to another object B in that T shares a property S with B and B satisfies another property P. Therefore, T satisfies P, too", or it may be expressed more formally using the following schema.

$$\frac{\begin{array}{l} S(B) \land P(B) \\ S(T) \end{array}}{P(T)}$$

Here, T will be called the ***target***, B the ***base***, S the ***similarity*** between T and B, and P the ***projected property***.

Nevertheless, the above description of the process of analogy is insufficient. Researchers studying analogy have come to recognize the necessity of revealing some implicit condition which influences the process but does not appear in the above schema. The importance of this has already been discussed in [2, 1]. The implicit condition to be satisfied by appropriate analogical factors, T,B,S, and P, can, formally, be characterized only by a given theory, written as $\mathcal{A}$. Thus, the following questions occur: In the case that analogical inference is done under a theory $\mathcal{A}$, 1) what knowledge shown by $\mathcal{A}$ is related to the process of analogy, and 2) how is this knowledge used in the process ?

The objective of this paper is to answer these questions. First, through a logical analysis of analogy, it is shown that, when an analogical inference is done under a theory $\mathcal{A}$, a particular form of rule must be involved in the theorems of $\mathcal{A}$ and that analogical inference is accomplished by two particular types of (generally non-deductive) conjectures. Then, a non-deductive inference is proposed, which is shown to be an adequate candidate to yield the conclusions of both these conjectures.

2 A Logical Analysis

2.1 Approach To A Seed of Analogy

We can understand analogical reasoning as follows:

(1) Example-based Information: "An object, x' (corresponding to a base), satisfies both properties S and P $(\exists x'.S(x') \wedge P(x'))$."

(2) Similarity-based Information: "Another object, x (corresponding to a target), satisfies a shared property S with x' $(S(x))$."

(3) Analogical Conclusion: "The object x would satisfy the other property P $(P(x))$."

Let $\mathcal{A}$ be a theory consisting of closed sentences of first order logic. Then,

"Analogy is to reason (3) from $\mathcal{A}$ together with (1)+(2)." (A)

Let this understanding be our starting point of analysis.

As analogy is not, generally, deductive, this starting point may, unfortunately, be expressed only as follows. In the notation of proof theory,

$$\mathcal{A}, (\exists x'.S(x') \wedge P(x')), S(x) \nvdash P(x). \tag{1}$$

As analogy, however, infers $P(x)$ from the premises, it implies that some knowledge is assumed in the premise part of (1). Let the assumed knowledge be $F(x)$, providing that it depends on the x in general. That is,

$$\mathcal{A}, (\exists x'.S(x') \wedge P(x')), S(x), F(x) \vdash P(x). \tag{2}$$

Thus, the essential information newly obtained by analogy is $F(x)$ in the above rather than the explicit projected property P[1]. Making $F_A(x)$ stand for the conjunction of the example-based information and $F(x)$, the above meta-sentence is transformed equivalently to

$$\mathcal{A} \vdash (\forall x F_A(x) \wedge S(x) \supset P(x)) \tag{3}$$

because $\mathcal{A}$ is closed. This implies that a rule must be a theorem of $\mathcal{A}$ and that the rule concludes any object which satisfies $F_A(x)$ to satisfy P when it satisfies S. This rule will be called the *analogy prime rule* (this rule will be specified in more detail later). Once F_A is satisfied, (by reason of $S(x) \supset P(x)$,) the analogical conclusion ("an object satisfies P") follows from the similarity-based information ("the object satisfies S). For this reason, F_A is called *analogy justification*.

Consequently, an object T which satisfies S is concluded to satisfy P from an analogy prime rule, by an analogy that assumes that T satisfies the analogy justification ($F_A(T)$). That is, our starting point (A) can be specified from two aspects.

"An analogical conclusion is obtained from an analogy prime rule together with example-based information and similarity-based information."

[1] Note that it includes a case, $F = P$.

(B)

"A non-deductive jump by analogy, if it occurs, is to assume that the analogy justification of the prime rule is satisfied." (C)

In the following part of this paper, analogy justification and non-deductivity will be further explored. Before begining an abstract discussion, it may be useful to see concrete examples of analogical reasoning. The next section introduces "target" examples of analogy to be clarified here.

2.2 Examples

Example1: Determination [2]. "Bob's car (C_{Bob}) and Sue's car (C_{Sue}) share the property of being 1982 Mustangs ($Mustang$). We infer that Bob's car is worth about \$3500 just because Sue's car is worth about \$3500. (We could not, however, infer that Bob's car is painted red just because Sue's car is painted red.)"
Example-based Information:

$$Model(C_{Sue}, Mustang) \wedge Value(C_{Sue}, \$3500), \tag{4}$$

Similarity-based Information:

$$Model(C_{Bob}, Mustang), \tag{5}$$

Example2: Brutus and Tacitus [1]. " Brutus feels pain when he is cut or burnt. Also, Tacitus feels pain when he is cut. Therefore, if Tacitus is burnt, he will feel pain."
Example-based Information:

$$(Suffer(Brutus, Cut) \supset FeelPain(Brutus)) \tag{6}$$

$$\wedge(Suffer(Brutus, Burn) \supset FeelPain(Brutus)) \tag{7}$$

Similarity-based Information:

$$Suffer(Tacitus, Cut) \supset FeelPain(Tacitus) \tag{8}$$

Example3: Negligent Student[2]. " When I discovered that one of the newcomers ($Student_T$) to our laboratory was a member of an orchestra club ($Orch$), remembering that another student ($Student_B$) was a member of the same club and he was often negligent of study ($Study$), I guessed that the newcomer would be negligent of study, too."

Example-based Information:

$$Member\_of(Student_B, Orch) \wedge Negligent\_of(Student_B, Study) \tag{9}$$

Similarity-based Information:

$$Member\_of(Student_T, Orch) \tag{10}$$

2.3 Logical Analysis: a rule as a seed of analogy

In treating analogy in a formal system, as the information of a base object being S and P is projected into a target object, it is desirable to treat such properties as *objects* so that we could avoid the use of second order language. As an example, the fact that Bob's car is a Mustang is represented by $Model(C_{Bob}, Mustang)$ rather than simply $Mustang(C_{Bob})$. In the remaining part, we use $K(x, S)$ and $U(x, P)$ for S and P. S as an *object* is *similarity-attribution* and P as an *object* is *projected-attribution*. Then, (3) is rewritten by

$$\mathcal{A} \vdash \forall x, s, p.\ F_A(x, s, p) \wedge K(x, s) \supset U(x, p), \tag{11}$$

considering the most general case that the analogy justification F_A depends on all of these factors.

Again, when 3-tuple $<$ object: X, similarity-attribution: S, projected-attribution: P $>$ satisfies an analogy justification F_A, the object X is conjectured to satisfy a projected property $\lambda x.U(x, P)$ (analogical conclusion) just because X has a similarity $\lambda x.K(x, S)$. That is, $F_A(x, s, p)$ can be considered such a condition that x could be concluded to be p from x being s by analogical reasoning.

[2] The author thanks Satoshi Sato for showing this challenging example.

Now, recalling that an analogical conclusion is obtained from an analogy prime rule with example-based information and similarity-based information, consider what information can be added by each information in relation to an analogy prime rule.

1) **Example-based Information:** This shows that there exists an object as a base which satisfies a similarity and a projected property ($\exists x'.K(x', S) \wedge U(x', P)$). It seems to be adequate that the base, B, satisfying $K(x', S)$ can also be derived to satisfy $U(x', P)$ from the prime rule. That is, the 3-tuple $< B, S, P >$ satisfies the analogy justification. Consequently, from arbitrary selection of an object as a base in this information, what is obtained from this information is $\exists x'.\ F_A(x', S, P)$.

2) **Similarity-based Information:** This shows that an object as a target, T, satisfies the same property S in the above. Just by this fact, an analogical conclusion is obtained, by assuming that the object satisfies F_A by some conjecture. That is, there exists some attribution p' and 3-tuple $< T, S, p' >$ satisfies F_A ($\exists p'.\ F_A(T, S, p')$).

3) **Analogical Conclusion:** With the above two pieces of information, an analogical conclusion, "T satisfies $U(x, P)$", is obtained from the analogy prime rule. Therefore, such 3-tuple $< T, S, P >$ satisfies F_A ($F_A(T, S, P)$).

In the above discussion, T, S, and P are arbitrary. Therefore, the following relation about the analogy justification turns out to be true.

$$\forall x, s, p.\ (\exists x'.\ F_A(x', s, p)) \wedge (\exists p'.\ F_A(x, s, p')) \supset F_A(x, s, p). \quad (12)$$

(12) is equivalent to being able to represent it as follows:

$$F_A(x, s, p) \ = \ G_{att}(s, p) \wedge G_{obj}(x, s), \quad (13)$$

where G_{att} is a sentence in which x does not occur free and G_{obj} is a sentence in which p does not occur free.

The point shown by this result is that any analogy justification can be represented by a conjunction in which variable x and variable p occur separately in different conjuncts.

By (11) and (13), the analogical prime rule can be defined as follows.

Definition 1 Analogy Prime Rule

A rule is called an analogy prime rule w.r.t. properties $K; U$, if it has the following form:

$$\forall x, s, p.\ G_{att}(s,p) \wedge G_{obj}(x,s) \wedge K(x,s) \supset U(x,p), \tag{14}$$

where $G_{att}(s,p)$, $G_{obj}(x,s)$, $K(x,s)$ and $U(x,p)$ are sentences in which neither of variables x, s, and p, other than variables in their own arguments, occurs free. □

In (14), $G_{att}(s,p)$ will be called *attribute-justification* and $G_{obj}(x,s)$ will be called *object-justification.*

Also, by the above discussion, the following two conjectures can be considered as causes which make analogy non-deductive.

- **Example-based Conjecture (EC):** An object shows a possible concrete combination of a similarity and a projected property, which allows the prime rule to be applicable to an adequate object.

$$\exists x.\ K(x,S) \wedge U(x,P) \mid\!\sim^{\mathcal{A}} G_{att}(S,P) \tag{15}$$

- **Similarity-based Conjecture (SC):** Just because an object satisfies S, application of the prime rule to the object is allowed.

$$K(x,S) \mid\!\sim^{\mathcal{A}} G_{obj}(x,S) \tag{16}$$

To summarize, a logical analysis of analogy could draw conclusions as follows.

Analogical reasoning is possible only if a certain *analogical prime rule* is a theorem of a given theory and the process of analogical reasoning can be divided into the following 3 steps: 1) the attribute-justification part of the rule is satisfied by EC from example-based information, 2) the object-justification part of the rule is satisfied by SC from similarity-based information, and, 3) from similarity-based information and the analogy prime rule specified by the two preceding steps, an analogical conclusion is obtained by deduction.

A question remains unclear, that is, what inference is EC and what SC? Though we cannot identify the mechanism underlying each of the conjectures, we can propose a (generally) non-deductive inference system as their candidates. The next section shows this.

2.4 Non-deductive Inference for Analogy

This section explores a type of generally non-deductive inference by which a conjecture G is obtained from a given theory $\mathcal{A}$ with additional information K.

Generally speaking, what properties should be satisfied by an (generally non-deductive) inference? It might be desirable that a non-deductive inference satisfies at least the following conditions. First, it subsumes deduction, that is, any deductive theorem is one of its theorems, because any deductive conclusion would be desirable. Secondly, any conclusion obtained by it must be able to be used deductively, that is, from such a conclusion, it should be possible to yield more conclusions using at least deduction. And thirdly, any conclusion obtained must be consistent with given information. We defines a class of inference system which satisfies the above three conditions.

Definition 2 *An inference system under a theory $\mathcal{A}$ (writing $\mid\!\sim^{\mathcal{A}}$) is conservative if the following conditions are satisfied. For any set of sentences $\mathcal{A}$ and K, and any sentence G and H,*

i) *Subsuming deduction: if* $\mathcal{A}, K \vdash G$ *then* $K \mid\!\sim^{\mathcal{A}} G$.

ii) *Deductive usefulness: if* $K \mid\!\sim^{\mathcal{A}} G$ *and* $\mathcal{A}, K, G \vdash H$ *then* $K \mid\!\sim^{\mathcal{A}} H$

iii) *Consistency: if* $K \mid\!\sim^{\mathcal{A}} G$ *and* $\mathcal{A} \cup K$ *is consistent then* $\mathcal{A} \cup K \cup \{G\}$ *is consistent.*

The following inference system is an example of a conservative system.

Definition 3 *G is a conjecture from $\mathcal{A}$ based on K by circumstantial reasoning (writing $K \mid\!\sim_{*}^{\mathcal{A}} G$), if for some set of clauses E,*

i) *E is a minimal set s.t.* $\mathcal{A}, E \vdash K$ *and* $\mathcal{A} \cup E$ *is consistent if* $\mathcal{A} \cup K$ *is consistent.*

ii) $\mathcal{A}, E \vdash G$.

Proposition 1 *If* $K \mid\!\sim^{\mathcal{A}} G$ *and* $K, G \mid\!\sim^{\mathcal{A}}_{*} H$*, then* $K \mid\!\sim^{\mathcal{A}} H$.

Corollary 1 *If* $K \mid\!\sim^{\mathcal{A}}_{*} G$*, then* $K \mid\!\sim^{\mathcal{A}} G$.

This corollary 1 shows that circumstantial reasoning is conservative, and proposition 1 (together with the corollary) shows that inference done by multiple applications of circumstantial reasoning is also conservative.

Circumstantial reasoning ($K \mid\!\sim^{\mathcal{A}}_{*} G$) [3] implies a very general and useful inference class in that so many types of inference used in AI can be considered as circumstantial reasoning. Deduction and abduction, for example, are obviously circumstantial reasoning. Inductive learning from examples is the case that $\mathcal{A}$ is empty in general, K is "examples" and G is inductive knowledge obtained by "learning"[4] [5].

Now, we assume that both EC and SC are circumstantial reasoning, but based on different information. Then, we can see analogical reasoning in more detail.

Let an analogy prime rule w.r.t. properties $K; U$ be a theorem of $\mathcal{A}$. Then, when example-based information, $K(B, S) \wedge U(B, P)$, is introduced, by circumstantial reasoning from the prime rule, some justification is satisfied, that is,

$$K(B, S) \wedge U(B, P) \mid\!\sim^{\mathcal{A}}_{*} G_{att}(S, P) \wedge G_{obj}(B, S), \tag{17}$$

which concludes a specified prime rule,

[3]Circumstantial reasoning is essentially equivalent to "abduction" + deduction [5, 6]. However, "abduction" has many definitions and various usages in different contexts, so we like to introduce a new term for the type of inference in definition 3 to avoid confusion.

[4]In this case, $G = E$ in definition 3, which implies that G is a minimal set to explain "example" K. Indeed, such minimality is very common in the field.

[5]Such a unified aspect of various reasoning in AI was pointed out by Koich Furukawa in a private discussion.

$$\forall x.\ G_{obj}(x, S) \wedge K(x, S) \supset U(x, P). \tag{18}$$

Even if similarity-based information $K(T, S)$ is introduced, to obtain analogical conclusion $U(T, P)$ by circumstantial reasoning, some information except the prime rule turns out to be needed in $\mathcal{A}$. And, both EC and SC are generally needed to accomplish analogical reasoning, which implies that multiple application of circumstantial reasoning is necessary. Even in such a case, circumstantial reasoning remains worthwhile (Proposition 1).

3 Classification of Analogy and Examples

Each EC and SC has two cases; deductive and non-deductive. According to this measure, analogical inference can be divided into 4 types. A typical example is shown in each class and explored.

3.1 deductive EC + deductive SC

Typical reasoning of this type was proposed by T.Davies and S.Russel [2]. They insisted that, to justify an analogical conclusion and to use information of the base case, a type of rule, called a *determination rule*, should be a theorem of a given theory. The rule can be written as follows:

$$\forall s, p.\ (\exists x'.\ K(x', s) \wedge U(x', p)) \supset (\forall x.\ K(x, s) \supset U(x, p)) \tag{19}$$

Example 1 (continued). In this example, the following determination rule holds under $\mathcal{A}$.

$$\forall s, p.\ (\exists x'.\ Model(x', s) \wedge Value(x', p)) \supset (\forall x.\ Model(x, s) \supset Value(x, p)) \tag{20}$$

This rule can be considered as the analogy prime rule, because

$G_{obj}(x, s) = \texttt{True}$,

$G_{att}(s, p) = (\exists x.\ Model(x, s) \wedge Value(x, p))$,

$K(x,s) = Model(x,s)$,

$U(x,p) = Value(x,p)$.

Moreover,

EC:

$$Model(C_{Sue}, Mustang) \wedge Value(C_{Sue}, \$3500) \vdash G_{att}(Mustang, \$3500), \quad (21)$$

SC:

$$Model(C_{Bob}, Mustang) \vdash G_{obj}(C_{Bob}, Mustang). \quad (22)$$

This illustrates that reasoning based on determination rules belongs to the "deductive EC + deductive SC" type and that it can also be done by circumstantial reasoning.

3.2 deductive EC + non-deductive SC

This type of analogical reasoning was explored by the author [1]. It was concluded that, once we assumed the following two premises for analogy, it seemed to be **an inevitable conclusion** that an analogy which infers $P(T)$ from $S(T), S(B)$, and $P(B)$ satisfies *the illustrative criterion*. And if an inference system satisfies the criterion, the system is called an *illustrative analogy*.

Premise 1: "Analogy is done by projecting properties (satisfied by a base) from the base onto a target."

Premise 2: "The target is not a special object."

Premise 2 is also assumed in this paper to be an arbitary selection of a target object. Premise 1 was translated as follows: $F_A(B)$, (where F_A is the justification in (3) and B stands for a base object) must be a theorem of $\mathcal{A}$, because it is essential in analogical reasoning to project $F_A(B)$ onto a target object T. That is, the non-deductive part in this reasoning is just SC which conjectures the property of the target object, and EC must be deductive.

Example 2 (continued). By illustrative analogy, a target is conjectured to satisfy properties used in an explanation of why a base satisfies a similarity. In this example, to explain why the base object, $Brutus$, satisfies the similarity that "if it is burnt, it feels pain", the following sentences must be in $\mathcal{A}$.

$$\forall x, s, i.\ Animal(x) \wedge Destructive(i) \wedge Suffer(x,i) \supset FeelPain(x), \tag{23}$$

$$Animal(Brutus),\ Destructive(Cut),\ Destructive(Burn) \tag{24}$$

From (23), the following follows:

$$\forall x, s, p.\ Animal(x) \wedge Destructive(s) \wedge Destructive(p)$$
$$\wedge(Suffer(x,s) \supset FeelPain(x)) \supset (Suffer(x,p) \supset FeelPain(x)), \tag{25}$$

which is an analogy prime rule, that is,

$G_{obj}(x,s) = Animal(x)$,
$G_{att}(s,p) = Destructive(s) \wedge Destructive(p)$,
$K(x,s) = Suffer(x,s) \supset FeelPain(x)$,
$U(x,p) = Suffer(x,p) \supset FeelPain(x)$.

$G_{att}(Cut, Burn)$ ("Both cut and burn are destructive") is a deductive theorem of $\mathcal{A}$ and a non-deductive conjecture, $G_{obj}(Tacitus, Cut)$ ("Tacitus is an animal"), is obtained by circumstantial reasoning from (23) based on the similarity-based information, $Suffer(Tacitus, Cut) \supset FeelPain(Tacitus)$.

3.3 non-deductive EC + deductive SC

As far as the author knows, this type of analogy has never been discussed. Example 3 seems to show this type of analogy.

Example 3 (continued). First, let us consider what we know from example-based information in this case. From the fact that a student ($Student_B$) was a member of the same club and often neglected study ($Study$), we could find that "the orchestra club keeps its members very busy ($BusyClub(Orch)$)" and that "activities of the club are obstructive to one's study ($Obstructive\_to(Orch, Study)$)". This implies that we knew some causal rule like "If it is a busy club and its activities are obstructive to something, then any member of the club neglects the thing."

$$\forall x, s, p.\ BusyClub(s) \land Obstructive\_to(p, s) \land Member\_of(x, s) \supset Negligent\_of(x, p), \tag{26}$$

And using this rule, we found the above information.

$BusyClub(Orch)$ and $Obstructive\_to(Orch, Study)$ are non-deductive conjectures and it can be obtained by circumstantial reasoning based on the above rule, which is just an analogy prime rule, as follows:

$G_{obj}(x, s) = \mathbf{True}$,
$G_{att}(s, p) = BusyClub(s) \land Obstructive\_to(p, s)$,
$K(x, s) = Member\ of(x, s)$,
$U(x, p) = Negligent\_of(x, p)$.

3.4 non-deductive EC + non-deductive SC

As an example of this type, we can take Example 2 again. We might know neither "Brutus is an animal" nor "Both cut and burn are destructive", which corresponds to the case that (24) is not in $\mathcal{A}$ (nor any deductive theorem of $\mathcal{A}$) in the previous Example 2. However, by circumstantial reasoning from (23) based on example-based information ("Brutus feels pain when he is cut or burnt"), "Both cut and burn are destructive" (and "Brutus is an animal") can be obtained, and based on similarity-based information ("Tacitus feels pain when he is cut"), "Tacitus is an animal" is obtained similarly to the previous example. Consequently, the analogical conclusion ("Tacitus would feel pain when he is burnt") is derived from (23) together with the above conjectures.

4 Conclusion and Remarks

- Through a logical analysis of analogy, it becomes clear that analogical reasoning is possible only if a certain *analogical prime rule* is a deductive theorem of a given theory. From the rule, together with *an example-based conjecture* and *a similarity-based conjecture*, *the analogical conclusion* is derived. A candidate is shown for a non-deductive inference system which yields adequately both conjectures.

- Results shown here are general and do not depend on particular pragmatic languages like the "purpose" predicate [3] nor some numeric similarity measure [8]. These results can be applied any normal deductive data bases (DDB) which consists of logical sentences. Interestingly, a method which discovers analogy prime rule from knowledge data-base CYC is, independently, explored [7]. Such methods would make analogical reasoning more common in DDB.

- An implementation system for this type of analogy is being developed. In such a system, the following are necessary: 1) to search for an analogy prime rule, and 2) to conduct circumstantial inference. Partial evaluation technology [4] will be used for 1) and methods for hypothetical reasoning [6] will be used for 2).

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