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Extended Algorithm for Causal Ordering Analysis (2)

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Abstract

This paper describes extended methods for causal analysis by means of causal ordering. The original algorithm proposed by Iwasaki[1] deals with constraints in the forms of equilibrium and of differential equations. All constraints are assumed to be always effective. However practical physical models are represented by constraints using inequalities, besides equations. Some constraints are conditional: they have preconditions which must be satisfied for the constraints to be applied.

Then we define causalities between variables derived from inequalities. We propose an algorithm that analyzes potential causalities that are derived from a set of conditional constraints. The algorithm uses only qualitative information of each constraint: the information concerning which variables appear in each constraint. It cannot specify which conditions are actually true because of the ambiguity of the qualitative value. However, the algorithm reasons all potential causal relations between variables by using sets of exclusive conditional constraints.

Key words: Causal ordering, Qualitative reasoning

1 Introduction

Many models for physical systems used in research related to qualitative reasoning can be represented in the form of constraints. A constraint is a declarative expression which contains multiple procedural functions: bidirectional dependencies between physical variables. However, in the actual problem solving based on the physical model, the notion of causality between variables, which is not explicitly described in the constraint expression, plays an important role. So an algorithm deriving the causal relations from the bidirectional constraint expression is necessary.

For this purpose, Iwasaki[1] proposed a theory which analyzes the device behavior by means of causal ordering. The algorithm deals with only constraints in the forms of equilibrium and of differential equations, all of which always operate.

However, in the currently available qualitative reasoning system [2,3,4], various types of formalism are used for knowledge representation. Qualitative models of practical physical objects can be represented by the following types of constraints, besides equations:

(a) inequalities

(b) conditional constraints: they are applied only if all of the preconditions are satisfied.

Then the original causal ordering algorithm is insufficient for analyzing practical physical models.

We intend to explore the causal ordering algorithm that can analyze causality between variables in a qualitative model represented by such types of constraints.

Section 2 outlines the original causal ordering algorithm. In section 3, we explain the extended causal ordering algorithm for inequalities. Section 4 describes the method for analyzing a causality derived from a set of conditional constraints. Section 5 discusses the efficiency of the causal analysis using qualitative information.

2 Review

We briefly outline the original causal ordering algorithm[1] for equilibrium equations. The basic principle of the algorithm is a determination of an order between variables given by solving simultaneous equations.

[Def.1] Causality in terms of causal ordering

If a set of N equations containing M variables have N undetermined variables, Ω, and (M-N) previously determined variables, Ψ, then we call the set self contained. And we call a self-contained set containing no self-contained subset a minimal complete set. The values of Ω in the minimal complete set can be determined by solving N simultaneous equations. Because the values of N newly determined variables, Ω, depend on the values of Ψ, we define causality in terms of causal ordering from Ψ to Ω.

The physical meaning of the causality is as follows. A set of physical laws described by the constraints in the minimal complete set operate in an environment specified by the states of the predetermined variables, and that set of laws determines the states of other physical variables. We define this dependency between the states of physical variables as a physical causality (see Fig.1).

The method uses only qualitative information: which variables appear in each constraint. That suits the analysis

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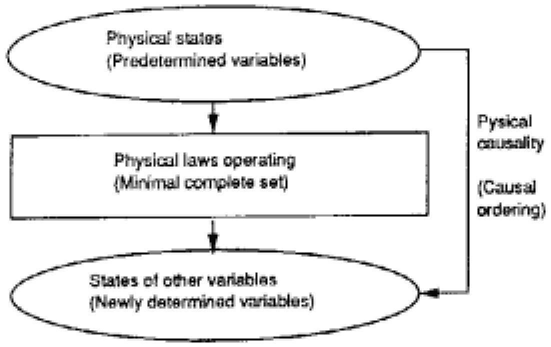

Fig.1 Physical meaning of causal ordering

of qualitative models where few parameters are quantitatively known. We will extend the algorithm in the same way.

3 Extended causal ordering algorithm for inequalities

Next we define causal relation derived from inequalities. There are two types of propagation of variable values through arithmetic reasoning. One is that values of variables are determined by solving simultaneous equations as mentioned in Def.1. The other is a limitation of the upper or lower bound of a variable through inequalities.

[Def.2] Interval causality in inequalities

If there is only one undetermined variable in an inequality, the values of other, determined, variables limit the upper or lower bound of the undetermined variable. We define this dependency of the undetermined variable on the determined variables as an interval causality.

Note that we do not distinguish between a limitation of the lower bound and a limitation of the upper bound of a variable. For example, assume that we know the value of X is already determined in inequality (1).

$$X * Y > 0 \qquad ...(1)$$

Then we can say that either of the upper or lower boundary of a variable, Y, is limited by the value of X. If we want to specify which of them is actually limited, we need the quantitative value of X by solving other simultaneous equations or inequalities. However it is, in general, less likely to be possible because we have less quantitative information of coefficients or parameters in qualitative models. So we define only a kind of interval causality in the analysis of inequalities.

4 Extended algorithm for conditional constraints

We explain an extended algorithm for analyzing conditional constraints. Hereafter we represent each conditional constraint as follows:

$$P_{j1}, ..., P_{jM_j} \Rightarrow C_j \qquad ...(2)$$

where $P_{j1}, ..., P_{jM_j}$ are preconditions, and C_j is a consequent of the constraint. P_{jk}, C_j are formulas in the form of equations and inequalities.

4.1 Definition of conditional causality

First we define causality derived from conditional constraints (2). Each conditional constraint is activated and constrains values of variables in the consequent only if all of the preconditions in its antecedent are satisfied. Because the values of all variables that appear in the antecedent must be previously determined, before we can evaluate the truth value of the antecedent, we define conditional causality derived from conditional constraints as follows:

[Def.3] Conditional causality

If values of all variables in an antecedent of a conditional constraint are determined, then we define dependency from the variables in the antecedents to the consequent of the constraint as a conditional causality.

There are two things to note. Firstly, the conditional causality is a dependency of a consequent of a conditional constraint on variables, not between variables. Secondly, in the causal analysis using qualitative information, the conditional causality does not mean that the antecedents of the conditional constraint is actually true and that the consequent holds true.

Consider the example shown in Fig.2, which is a physical model of water and steam enclosed in a piston and cylinder system. When T and V are directly controllable, a set of simplified constraints describing the qualitative relations between V, T and P are following:

$$\left.\begin{array}{l} E_a1; T = t_a \\ E_a2; P = p_a \\ E_a3; T \leq t_b \Rightarrow V = k_2 * T + k_3 \\ E_a4; T > t_b \Rightarrow P * V = k_1 * T \end{array}\right\} ...(3)$$

Where k_1, k_2, and k_3 are constants. As the value of T is determined by equation E_a1, we can find a conditional causality from T to the consequent of E_a3. However, we cannot say whether the precondition of $E_a3, T \leq t_b$, is actually satisfied, because we do not have the quantitative value of T determined by E_a1. This ambiguity originates from the use of qualitative information.

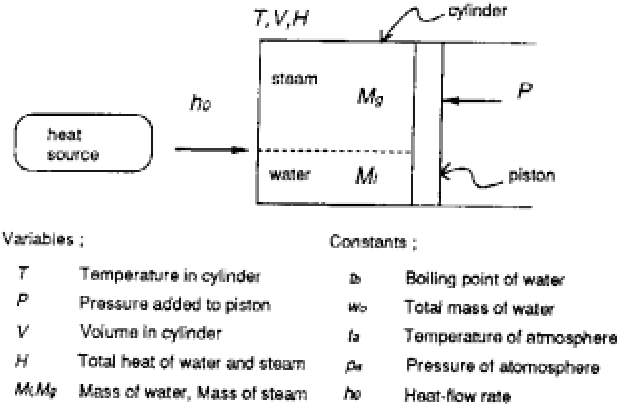

Fig.2 Physical model of water and steam enclosed in a cylinder and piston system

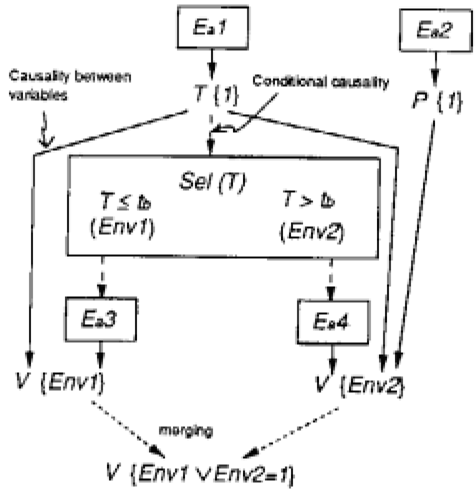

Fig.3 Potential cousal graph of constraints (3)

4.2 Algorithm for analyzing potential causality

Next we will explain a potential causality derived from a set of exclusive conditional constraints. Many physical laws and behavioral characteristics of components are usually described by a set of constraints that operate on exclusive environments. For such exclusive conditional constraints, we can analyze potential causality between variables by enumerating all causal relations in each context, or combination of environments.

In the example of (3), E_a3 and E_a4 form a set of exclusive constraints, because their preconditions are exclusive of each other. When the value of T is determined by E_a1, either of the two conditional constraints is activated no matter what is the determined value of T.

We will create a conceptual selector switch $Sel(T)$, which branches out to a set of exclusive environments, $Env1(T \leq t_b)$ and $Env2(T > t_b)$. If the value of T is determined, $Sel(T)$ becomes active and activates all the environments under it.

Each conditional constraint is activated if all of its environments corresponding to its preconditions are active. E_a3 is activated on $\{Env1\}$, and so E_a4 is on $\{Env2\}$. Causal relations between variables can be analyzed for each context by using the active constraints. We can acquire causal relation from the variable T to V by using equation E_a3 on the context, $\{Env1\}$, because E_a3 is active only on $\{Env1\}$, although the value of the variable T is universally (in all environments) determined. A causality from P and T to V is acquired by using E_a4 on $\{Env2\}$ as well (see Fig.3).

We call these two causal relations potential, because each of them can be valid only in its active context. It is possible that the value of V is determined, on $\{Env1 \vee Env2 = 1\}$, although the constraint which determines the value of V is different for each environment. So we call V is universally active, on $\{1\}$. (Here $\{1\}$ denotes a universal set.)

4.3 Iteration method for analyzing potential causality

We will introduce an iteration method for preventing a deadlock caused by a loop of conditional causality between sets of conditional constraints. Consider the following set of simplified constraints describing phase change between water and steam in the model of Fig.2 when heat, H, is applied to the water and steam;

$$\left.\begin{array}{l}
E_b1; dH/dt = h_0 \\
E_b2; T < t_b \Rightarrow M_g = 0 \\
E_b3; T < t_b \Rightarrow M_l = w_0 \\
E_b4; T = t_b \Rightarrow m^+(M_g, H) \\
E_b5; T = t_b \Rightarrow m^-(M_l, H) \\
E_b6; T > t_b \Rightarrow M_g = w_0 \\
E_b7; T > t_b \Rightarrow M_l = 0 \\
E_b8; M_l > 0 \wedge M_g = 0 \Rightarrow m^+(T, H) \\
E_b9; M_l > 0 \wedge M_g > 0 \Rightarrow dT/dt = 0 \\
E_b10; M_l = 0 \wedge M_g > 0 \Rightarrow m^+(T, H)
\end{array}\right\} \dots(4)$$

Here $m^+(X,Y)$ ($m^-(X,Y)$) is a monotonically increasing(decreasing) functional relationship between X and Y. When X increases, Y increases(decreases), and vice versa.

Constraints from E_b2 to E_b7 are activated by the determination of value of T. And they are the only constraints which can determine the values of M_l and M_g. Only E_b8, E_b9 and E_b10, which are activated by the variables M_l and M_g, can determine the value of T. The conditional causality between $[M_l, M_g]$ and $[T]$ makes a loop. Since all values of the variables, M_l, M_g and T, are undetermined at the beginning of causal analysis, no selector or conditional constraint can be active. As there are no active constraints, the

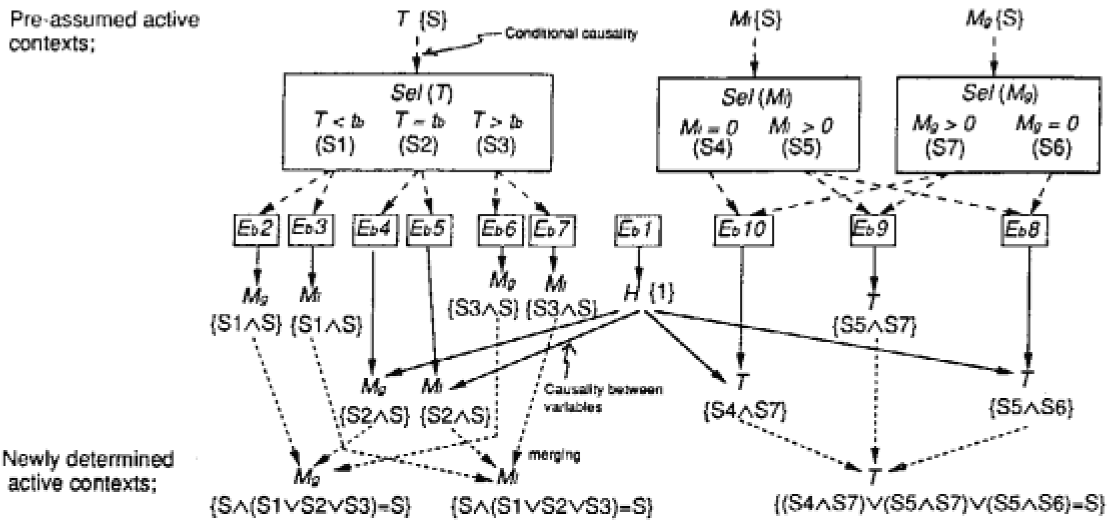

Fig.4 Causal graph of constraints (4) --- (converged state at the end of third iteration)

Number of iteration	pre-assumed T	pre-assumed M_g	pre-assumed M_l		determined T	determined M_g	determined M_l
1	{1}	{1}	{1}	→	{S}	{1}	{1}
2	{S}	{1}	{1}	→	{S}	{S}	{S}
3	{S}	{S}	{S}	=	{S}	{S}	{S}

Table 1 Changes of active contexts of variables for iteration

values of M_l, M_g and T are not determined in any contexts. So the causal analysis comes to a deadlock.

In an actual physical system, each variable has a certain value in an arbitrary environment. Even if there is a loop of conditional causality between sets of constraints, quantitative values of some variables specify which conditional constraints actually operate and determine the states of the other variables. So we can find dependency between variables.

We introduce the following iteration method for avoiding a deadlock of causal analysis. First, we assume that all variables in antecedents of all conditional constraints are universally active. Next, selectors and constraints are activated using the pre-assumed active contexts of the variables. Then all of the potential causalities in each context are analyzed, and the active contexts of the variables are acquired. This procedure is iterated until the active contexts of all variables in the antecedents converge.

The changes of active contexts of the variables for the iteration procedure in the example (4) are shown in Table 1. In the first iteration, $Sel(T)$,$Sel(M_l)$ and $Sel(M_g)$, are universally activated by assuming T, M_l and M_g are universally active. Their active contexts are propagated to Envs under the selectors, $S1,...,S7$. They activate all of the conditional constraints, $E_b2,...,E_b10$, in the environments corresponding to the preconditions in their antecedents. And the active conditional constraints specify the causalities between variables. At the end of the first iteration, M_l and M_g are active in all environments. However the newly determined active context of T,

$$S = \{(S4 \wedge S7) \vee (S5 \wedge S7) \vee (S5 \wedge S6)\}$$

is not equal to its pre-assumed active context. The newly determined active context of T is substituted to the pre-assumed active context of T in the second iteration, and the procedure is iterated. In this example, all of the active contexts of T, M_l and M_g are converged to S after the third iteration. Then the causal analysis is terminated and we can acquire the potential causality of the set of constraints (4) as Fig.4.

4.4 Total extended algorithm for causal analysis

Now we can totally conclude the method for analyzing potential causality, including the case when the conditional causality between sets of conditional constraints makes a loop, as follows:

[Algorithm 1] Algorithm for analyzing potential causality

(c1) For each set of exclusive preconditions, $[P_{i1}, ..., P_{iL_i}]$, in antecedents of conditional constraints, create a selector switch

$$Sel_i(\Phi_i)[Env(P_{i1}), ..., Env(P_{iL_i})]$$

where, Φ_i is a set of variables appearing in the exclusive preconditions.

(c2) For all variables $\Phi = \Phi_1 \cup ... \cup \Phi_s$, assume that they are universally active. (These pre-assumed active contexts of variables are used only for activating selectors and constraints in steps (c3) and (c4).)

(c3) For each conditional selector switch, $Sel_i(\Phi_i)$, all active contexts of the variables in Φ_i are combined into an active context of the selector:

$active(Sel_i(\Phi_i)) = \bigwedge active(\Phi_i)$.

Add conditional causality:

$$\Phi_i \longrightarrow Sel_i(\Phi_i) \text{ on } active(Sel_i(\Phi_i))$$

And this active context is propagated to each environment under the selector:

$$active(Env(P_{ih})) = Env(P_{ih}) \wedge active(Sel_i(\Phi_i))$$

(c4) For each conditional constraint, C_j, shown as (2), calculate its active environment,

$$active(C_j) = active(Env(P_{j1})) \wedge ... \wedge active(Env(P_{jM_j})).$$

Each constraint without preconditions is universally active. Add conditional causality:

$$Env(P_{j1}), ..., Env(P_{jM_j}) \longrightarrow C_j \text{ on } active(C_j).$$

(c5) Minimal complete sets defined as Def.1 are searched from the set of consequents of all active constraints. For each set of N simultaneous active equations, $C = [C_1, ..., C_N]$, containing M variables, the active context is:

$$active(C) = active(C_1) \wedge ... \wedge active(C_N).$$

If there is a sub-context of $active(C)$, where values of $(M - N)$ variables, Ψ, are determined and the rest N variables, Ω, are undetermined, then Ψ and C determine the values of Ω. The newly determined context of the variables, Ω, is:

$$active(\Omega) = active(C_1) \wedge ... \wedge active(C_N) \wedge active(\Psi).$$

Acquired potential causality is:

$$\Psi \text{ and } C_1, ..., C_N \longrightarrow \Omega \text{ on } active(\Omega).$$

Note that in the procedure searching minimal complete sets, every constraint can be used only once in each context.

Potential interval causality defined in Def.2 is acquired as well.

(c6) For each variable, merge all of the determined environments acquired in step (c5) to its currently determined active context.

If for each variable in Φ, the currently determined active context and the pre-assumed active context are the same, then the potential causalities are acquired as the final state of the procedure.

else go to step (c3) after substituting the currently determined active contexts to the pre-assumed active contexts in the next iteration, for all variables in Φ.

5 Summary and discussion

We extended the causal ordering algorithm until it can analyze causalities of a qualitative model of a practical physical object.

(a) The extended algorithm can analyze causalities of inequalities.

(b) It can also analyze conditional constraints: it reasons all possible potential causal relations between variables by using sets of exclusive conditional constraints.

Two problems remain that originate from the ambiguity of qualitative information. First, the method cannot specify which potential causality actually holds, as discussed in section 4.

Second, the method cannot eliminate the invalid context. In the example of Fig.4, the combination of environment, $S4(M_i = 0) \wedge S6(M_g = 0)$, is invalid if the value of w_0 is positive. However as the algorithm does not use the qualitative value of w_0, it cannot eliminate such an invalid context in the calculation of the active context.

In the actual model-based problem solving [5,6], both a qualitative model and a quantitative model are used. In the research of diagnosis [5], a qualitative model is used for generating candidates of faults and a quantitative model is used for verification of the candidate. So if we regard the method as a tool for generating comprehensive causal relations, we can expect the algorithm to become a powerful tool for model-based problem solving.

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