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Merging Situations

by

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## Merging situations\*

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### 1. Introduction

In applying or implementing situation theory it is often useful to use a notion of merging the information supported by two situations which is related to computational notions of unification. There is a trivial notion of merging two situations which might be defined as follows:

The result of merging two situations  $s_1$  and  $s_2$  which are consistent with each other is the situation  $s_1 \cup s_2$  where

$$s_1 \cup s_2 \models \sigma$$

just in case either  $s_1 \models \sigma$  or  $s_2 \models \sigma$ . ( $s_1 \cup s_2$  is undefined if  $s_1$  and  $s_2$  are not consistent with each other.)

Two situations  $s_1$  and  $s_2$  are consistent with each other just in case there is no  $\sigma$  such that either  $s_1 \models \sigma$  or  $s_2 \models \sigma$  whose dual is also supported by either  $s_1$  or  $s_2$ .

This simple definition of merging is, however, not adequate to correspond to computational notions of unification since it is in no way recursive. That is, there is nothing in this definition of NON-

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RECURSIVE MERGE which will require the merging of situations which are arguments in the possible facts supported by situations being merged.

This technical note will attempt to make precise a notion of RECURSIVE MERGE and then show how graph unification can be defined as a special case of this.

## 2. Compact situations

We will first define a preliminary notion COMPACT SITUATION. First a little notation. We will use  $\sigma(\vec{s})$  to represent a possible fact where  $\vec{s}$  represents the assignment of situations to argument roles of  $\sigma$ 's relation such that there are no situations assigned to arguments roles in  $\sigma$  except as indicated by  $\vec{s}$ . We will use  $\sigma(\vec{s} / \vec{s}')$  (where  $\vec{s}$  and  $\vec{s}'$  assign situations to the same roles) to represent a fact like  $\sigma(\vec{s})$  except that the assignment  $\vec{s}'$  is used in place of  $\vec{s}$ .

A situation *sit* is COMPACT iff

for all  $\sigma(\vec{s})$  such that  $sit \models \sigma(\vec{s})$   
 there is no  $\vec{s}' \neq \vec{s}$  such that  $sit \models \sigma(\vec{s} / \vec{s}')$

### Examples

In all examples in this note we specify all the facts supported by the situations.

#### (1) A compact situation

$s \models \langle \text{see}, a, b \rangle$

$s \models \langle \text{see}, a, c \rangle$

$s \models \langle \text{see}, a, s_1 \rangle$

Here there is only one fact which has a situation as argument. Thus there are no facts which are exactly similar except that they have different situations playing the same arguments roles.

(2) A compact situation

$$s \models \langle \text{see}, a, b \rangle$$
$$s \models \langle \text{see}, a, s_1 \rangle$$
$$s \models \langle \text{see}, b, s_2 \rangle$$

This situation is compact because, although it supports two facts which have possibly different situations playing the same roles, the facts are not otherwise exactly similar, one having the argument  $a$  where the other has the argument  $b$ .

(3) A situation which is not compact

$$s \models \langle \text{see}, a, s_1 \rangle$$
$$s \models \langle \text{see}, a, s_2 \rangle$$

where  $s_1 \neq s_2$

This situation is not compact because it supports facts which are exactly similar except that they have distinct situations which play the same role. The intuition we wish to capture is that this situation could be made compact if  $s_1$  and  $s_2$  can be merged to  $s_3$  and we define a new situation  $s'$  such that

$$s' \models \langle \text{see}, a, s_3 \rangle$$

Note that there will not be any such situation if  $s_1$  and  $s_2$  fail to merge because they are inconsistent with each other. Thus attempting to compact a situation might reveal an inconsistency which was hidden in the original situation.

### 3. Merging compact situations

We will define our notion of merge on compact situations and also parameterize the definition with respect to constraints  $C$ .

A situation  $s$  is the merging of compact situations  $s_1$  and  $s_2$  with respect to  $C$  (in symbols  $s_1 \cup_C s_2$ ) if

either  $s = s_1 = s_2$

or  $s$  supports the least number of facts such that it meets condition  $C$ , is consistent and

$$\begin{aligned} \text{a. } s \models \sigma(\vec{s}_a \cup_C \vec{s}_b) \\ \text{if } s_1 \models \sigma(\vec{s}_a) \\ \text{and } s_2 \models \sigma(\vec{s}_b) \end{aligned}$$

(where  $\cup_C$  represents pointwise merging with respect to  $C$ )

$$\begin{aligned} \text{b. otherwise } s \models \sigma^+ \\ \text{if } s_1 \models \sigma \text{ or } s_2 \models \sigma \end{aligned}$$

where  $\sigma^+$  is the result of substituting  $s$  for  $s_1$  and  $s_2$  in  $\sigma$  and  $s_a \cup_C s_b$  for any  $s_a$  and  $s_b$  merged as a result of clause (a).

#### Example

(1) Two utterance situations where merging succeeds

$$u_1 \models \langle \text{case}, u_1, \text{dat} \rangle$$

$$u_2 \models \langle \text{case}, u_2, \text{dat} \rangle$$

$$u_1 \cup_C u_2 \models \langle \text{case}, u_1 \cup_C u_2, \text{dat} \rangle$$

$$\begin{aligned} C = \text{if } s \models \langle \text{case}, s, c \rangle \text{ and } s \models \langle \text{case}, s, c' \rangle \\ \text{then } c = c' \end{aligned}$$

(2) Two utterance situations where merging succeeds

$$u_1 \models \langle \text{case}, u_1, \text{dat} \rangle$$

$$u_2 \models \langle \text{case}, u_2, \text{acc} \rangle$$

$$u_1 \cup_C u_2 \models \langle \text{case}, u_1 \cup_C u_2, \text{dat} \rangle$$

$$u_1 \cup_C u_2 \models \langle \text{case}, u_1 \cup_C u_2, \text{acc} \rangle$$

$$C = \{\} \text{ (no constraints)}$$

This merging succeeds under the linguistically unnatural assumption that  $C$  is empty.

(3) Two utterances where merging fails

$$u_1 \models \langle \text{case}, u_1, \text{dat} \rangle$$

$$u_2 \models \langle \text{case}, u_2, \text{acc} \rangle$$

$$C = \text{if } s \models \langle \text{case}, s, c \rangle \text{ and } s \models \langle \text{case}, s, c' \rangle \\ \text{then } c = c'$$

This merge fails because there is no situation which meets the clauses (a.) and (b.) of the definition of merge and in addition meets the constraint  $C$ .

#### 4. Graph Unification

A DIRECTED GRAPH is a situation  $s$  such that

1) for all  $\sigma$ ,  $s \models \sigma$  implies

either there is some  $r, v$  such that

$$\sigma = \langle r, s, v \rangle \text{ "r has value v in s"}$$

or  $\sigma = \langle \text{linked}, r_1, s, r_2, s' \rangle$  "the attributes  $r_1$  in  $s$  is  
linked to the attribute  $r_2$  in  
 $s'$ "

2) if  $s \models \langle r, s, v \rangle$  and  $s \models \langle r, s, v' \rangle$  then  
 $v = v'$

3)  $s \models \langle \text{linked}, r_1, s, r_2, s' \rangle$  iff  
 $s' \models \langle \text{linked}, r_2, s', r_1, s \rangle$

4) if  $s \models \langle \text{linked}, r_1, s, r_2, s' \rangle$  then  
 $s \models \langle r_1, s, v \rangle$  iff  $s' \models \langle r_2, s', v \rangle$

Let the condition  $G$  be the conjunction of (1) and (2). Then the GRAPH UNIFICATION of  $s_1$  and  $s_2$  is  $s_1 \cup G s_2$ .

Remark: if  $s_1 \cup G s_2$  is defined then  $s_1, s_2$  and  $s_1 \cup G s_2$  are all graphs.

## Examples

(1)

If

$s_1 \models \langle \text{subj}, s_1, s_2 \rangle$

$s_1 \models \langle \text{verb}, s_1, s_3 \rangle$

$s_2 \models \langle \text{pers}, s_2, \text{first} \rangle$

$s_3 \models \langle \text{pers}, s_3, \text{first} \rangle$

$s_4 \models \langle \text{subj}, s_4, s_5 \rangle$

$s_5 \models \langle \text{subj}, s_5, \text{sg} \rangle$

then

$s_1 \cup_G s_4 \models \langle \text{subj}, s_1 \cup_G s_4, s_2 \cup_G s_5 \rangle$

$s_1 \cup_G s_4 \models \langle \text{verb}, s_1 \cup_G s_4, s_3 \rangle$

$s_2 \cup_G s_5 \models \langle \text{pers}, s_2 \cup_G s_5, \text{first} \rangle$

$s_2 \cup_G s_5 \models \langle \text{num}, s_2 \cup_G s_5, \text{sg} \rangle$

Note:  $s_2$  and  $s_3$  correspond to the same attribute-value matrix but they are nevertheless distinct situations since they support different facts. The finer granularity is achieved by including the argument for the situation itself in the facts. This assumes a version of situation theory which makes these distinct, which is not the case, for example, in the model suggested by Barwise.

(2)

The antecedent is like (1) except that instead of

$s_1 \models \langle \text{verb}, s_1, s_3 \rangle$

we have

$s_1 \vdash \langle \text{verb}, s_1, s_2 \rangle$

i.e. the `subj` and `verb` attributes share their value. The consequent is like (1) except that instead of

$s_1 \cup_G s_4 \vdash \langle \text{verb}, s_1 \cup_G s_4, s_3 \rangle$

we have

$s_1 \cup_G s_4 \vdash \langle \text{verb}, s_1 \cup_G s_4, s_2 \cup_G s_5 \rangle$

(3)

If

$s_1 \vdash \langle \text{linked}, \text{subj}, s_1, \text{verb}, s_1 \rangle$

$s_2 \vdash \langle \text{subj}, s_2, s_3 \rangle$

then

$s_1 \cup_G s_2 \vdash \langle \text{linked}, \text{subj}, s_1 \cup_G s_2, \text{verb}, s_1 \cup_G s_2 \rangle$

$s_1 \cup_G s_2 \vdash \langle \text{subj}, s_1 \cup_G s_2, s_3 \rangle$

$s_1 \cup_G s_2 \vdash \langle \text{verb}, s_1 \cup_G s_2, s_3 \rangle$

**Note:** the relation `linked` is introduced just to treat those cases where in the graph representation two paths are linked but no value is provided. If we exclude such cases then the characterization of a directed graph becomes much simpler since we could leave out the second disjunct of clause (1) and clauses (3) and (4). Note that the linking suggested here is more general than is normally assumed in graph unification since it enables us to link an attribute in one graph to an attribute in a distinct graph.