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(PRELIMINARY REPORT)

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Abstract

Circumscription proposed by McCarthy is one of the most hopeful formalization of nonmonotonic aspects of commonsense reasoning. It has several versions, however, they are all proposed for *denotative* minimization of predicates, that is, circumscription minimizes the extension of predicates. On such treatment, this paper considers three problems; absence of abnormal things, a limitation on equality and formalization of the unique name assumption. And it proposes a solution for them by presenting a *connotative* treatment of circumscription. This treatment is based on the idea of circumscribing predicates connotatively, that is, minimizing the set of names denoting objects which satisfy certain predicates.

1. Introduction

Consider the following situation.

Situation: "A man in a living room hears someone knocking on the front door. He knows that if someone knocks it is normally a man, however, he remember an exception of it, Tweety (he was a woodpecker). He is very tired. So he tries to confirm without moving and asks 'Who is it?'. But the visitor does not answer..."

In this situation, we expect that he would conclude the anonym is a man after all and would start to walk to the door reluctantly. Because he knows that if someone knocks it is normally a man and he does not have any available information which contradicts the conclusion.

McCarthy proposed a way to represent facts about what is "normally" the case. The key idea is minimization of abnormality and, to minimize some predicates, he propose a form called circumscription[6,7].

For instance, the situation is expressed as follows. (To clarify our arguments, it is simplified.)

Example 1. Let a sentence, A, be

$$\forall x.(\text{Knocks}(x) \wedge \neg \text{Ab}(x) \supset \text{Man}(x)) \quad (1)$$
$$\wedge \quad \text{Knocks}(\text{anonym}) \quad (2)$$
$$\wedge \quad \text{Ab}(\text{tweety}) \quad (3)$$
$$\wedge \quad \neg \text{Man}(\text{tweety}) \quad (4)$$

and, according to the idea of circumscription, we minimize (circumscribe) a predicate, Ab, with allowing to vary a predicate, Man. Because we want to think that each object is not abnormal unless available information prevents and we want to know whether the object is a man or not. However, there are three unsatisfied points on such treatment of circumscription.

1) Absence of abnormal things
The circumscription of Ab with allowing to vary Man in A yields

$$\forall x.(\text{Ab}(x) \equiv x = \text{tweety}). \quad (5)$$

That is, the circumscription tells that Tweety is the only abnormal. This result is too strong and somewhat unnecessary. What we want is the conclusion that the anonym is a man, that is, we want simply to think that the anonym is not abnormal if it is consistent with the given sentence, but not to know what is abnormal. To obtain the intended conclusion, why do we have to think that the only thing satisfies the property, Ab?

2) A limitation on equality
The most important and serious problem is on a limitation of circumscription on equality. We showed that circumscription (of any predicates with any predicates or functions allowing to vary) can not yield a new fact that varies the least cardinal number of domains of models of a given sentence if there exists a model whose domain consists of finite objects[1]. Returning to the Example 1, from A and the result of circumscription (5) we can obtain

$$\text{anonym} \neq \text{tweety} \supset \text{Man}(\text{anonym}). \quad (6)$$

However, we can never obtain the expected fact that the anonym is a man, Man(anonym). Because if circumscription could yield the fact then the circumscription changes the least cardinality from 1 to 2. (Notice that there exist a model of A whose domain consists of one object. So the least cardinality of A is 1. But, in a model of A where Man(anonym) holds, anonym$\neq$tweety holds, too. So the least cardinal number of such models is 2.) This contradicts. This shows that without additional axioms to A circumscription could never yield our intended results.

3) Formalization of the unique name assumption

Readers may think of the use of the unique name assumption for the above problem. However, we do not presently know satisfiable formalization of the unique name assumptions. It is closely related to the limitation mentioned above.

Reiter introduced the idea of the "unique name assumption (or, hypothesis)"[8], that is, distinct names denote distinct objects unless available facts imply that those objects are equal. Some approaches to formalization of the unique name assumption have been taken, however, we are not satisfied with them yet. For instance, McCarthy's solution [7] uses the language which involves the names themselves as the only objects. So all assertion about objects must be expressed as assertions about names. This may be considered unnatural[5]. Lifschitz presented another solution, which provides (finitely many) symbols for both names and denotation and introduced an unary function from names to objects. His solution also involves axioms which represent that the names are distinct each other. Notice that these axioms provides a sufficient number of objects to yield facts on inequality under the limitation mentioned in 2). Yet, his solution is insufficient by reason of the following two points; i) his solution does not express the unique name assumption for infinite names. (For the infinite names, infinite axioms would be needed.) and ii) if the given axioms involves the domain closure axiom the solution cannot be applied generally. Given a sentence with a domain closure axiom,

$$P(a) \land P(b) \land \forall x.(x = a), \tag{7}$$

we cannot provide distinct names for a and b, because the domain of any model of the sentence consists of one object.

This paper attempts to solve these problems. Our approach is to present a way to circumscribe predicates connotatively, whereas each version of circumscription proposed so far circumscribes predicates denotatively. Our result is called the connotative circumscription. It minimizes the set of names denoting objects which satisfy certain predicates. Intuitively, the sentence expresses an idea; "*the names that can be shown to denote the objects satisfying a certain property P from certain facts A are all the names denoting objects which satisfy P*". We show this in the next section.

2. Connotative Circumscription

Our solution is based on the following three treatments.

(i) To enable to vary the cardinal number of models, we must make domains controllable. For this purpose, at least, we need to introduce an unary predicate representing the whole domain of a given sentence.

(ii) To yield inequalities, we must provides enough many distinct symbols. Moreover, we must to make distinct names. For this purpose, we use a number theory and express infinite names with finite sentences.

(iii) To connect names with their objects, we assign distinct numbers as names to each grounded terms (i.e. terms without variables) using a unary function.

Details are as follows.

Let L_E be the external language with equality which is used for representation of the concerning universe, that is, let a given sentence A be a sentence of L_E, and L_I the internal language which is disjoint with L_E. Let L_I involve the following unary predicates; D, N, C, and the following functions (constants are considered 0-ary functions); 0 (for zero), s (for successor), + (for addition), · (for multiplication) and $\uparrow$ (for exponentiation), and also involve an unary function ε (for assignment function of objects to numbers). Our approach uses the language $L = L_E \cup L_I$.

(i) To represent the whole domain of a given sentence, the sentence is relativized with respect to a certain predicate. Such relativizing is based on the works by Davis [2], McCarthy [6] and Etherington [3]. Davis and McCarthy proposed methods for it and Etherington revised them a bit for the case of the relativized sentence is ∀ or ∀∃ theories having no object constants.

Let B be a sentence, Φ an unary predicate constant, F a tuple of function constants and $B^{*\Phi;F}$ be the conjunction of sentences

$$\exists x.\Phi(x) \wedge B^{\Phi} \wedge \mathrm{Func}^{\Phi}(F), \tag{8}$$

where B^{Φ} is formed by replacing each universal quantifier '$\forall x. \cdots$' in B by '$\forall x.(\Phi(x) \supset \cdots)$' and each existential quantifier '$\exists x. \cdots$' by '$\exists x.(\Phi(x) \wedge \cdots)$'. $\mathrm{Func}^{\Phi}(F)$ is the conjunction of sentences $\forall x_1,\ldots,x_n.(\Phi(x_1) \wedge \ldots \wedge \Phi(x_n) \supset \Phi(f(x_1,\ldots,x_n)))$ for each n-ary functions in F (for 0-ary functions, c, $\Phi(c)$).

Then we use the following sentence

$$\text{(a)} \qquad A^{*D;F}, \tag{9}$$

where F consists of functions in L_E. (Notice A is in L_E and D is in L_I.)

(ii) To represent a number theory, we use the following sentence.

$$\text{(b)} \qquad A_N{}^{*N;G}, \tag{10}$$

where A_N is a sentence

$$\begin{aligned}&\forall x.(s(x) \neq 0) \wedge \forall x,y.(s(x)=s(y) \supset x=y)\\ &\wedge \forall x.(x+0=x) \wedge \forall x,y.(x+s(y)=s(x+y))\\ &\wedge \forall x.(x\cdot 0=0) \wedge \forall x,y.(x\cdot s(y)=(x\cdot y)+x)\\ &\wedge \forall x.(x \uparrow 0=1) \wedge \forall x,y.(x \uparrow s(y)=(x \uparrow y)\cdot x),\end{aligned} \tag{11}$$

and G is $0,s,+,\cdot,\uparrow$.

In the rest part of this paper, we abbreviate s(0), s(s(0)), ... as 1, 2, ... respectively and $x \uparrow y$ as x^y. We call 0,1,... (natural) numbers.

(iii) To assign distinct numbers to each grounded terms, we need preliminaries.

For each $n \geq 1$, let a n-ary recursive function $,<>^n$, satisfy following conditiones;

$<>^n: N^n \rightarrow N.$

For all numbers $a_1,\dots,a_n$, b, there exists a 2-ary recursive function, β, such that

if $<a_1,\dots,a_n>^n = b$,
$\beta(b,0) = n$ and $\beta(b,i) = a_i \quad (1 \leqq i \leqq n)$ ■

For example, $<a_1,\dots,a_n>^n \equiv 2^n \cdot 3^{a_1} \cdot \cdots \cdot Pr(n+1)^{a_n}$, where $Pr(n)$ is the n-th prime number.

We take such a $<>^n$ and assign an number, $\ulcorner f \urcorner$, to each function, f, in F distinctly (that is, $N(\ulcorner f \urcorner)$ holds). Moreover, we use another abbreviation $`f(x_1, \dots, x_n)$ of $<\ulcorner f \urcorner, x_1, \dots, x_n>^{n+1}$. (We can consider 'f an n-ary function from N^n to N.) Notice that for each grounded term we can obtain a distinct natural number as its name by applying recursively these functions which correspond to each functions in the term.

Then we introduce the sentence

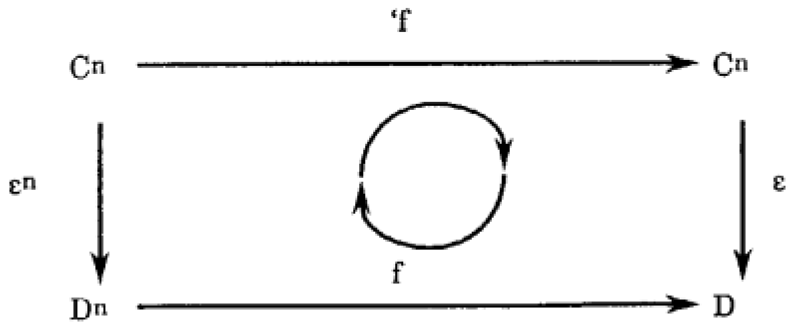
$$\text{(c)} \qquad \varepsilon\text{-axiom}(C;D;\varepsilon;F), \qquad (12)$$

which is the conjunction of a sentence $\forall x.(C(x) \supset D(\varepsilon(x)))$ and sentences

$$\forall x_1,\dots,x_n.(\, C(x_1) \land \dots \land C(x_n) \supset$$
$$f(\varepsilon(x_1),\dots,\varepsilon(x_n)) = \varepsilon(`f(x_1,\dots,x_n)) \land C(`f(x_1,\dots,x_n))\,)$$

for each n-ary function in F (for 0-ary functions, c, $c = \varepsilon(`c) \land C(`c)$).

That is, the following diagram commutes.



Now, we can introduce *connotative circumscription*. Connotative circumscription expresses minimization of the set, C, of names, x, denoting objects, $\varepsilon(x)$, which satisfy certain predicate.P. That is, it is to minimize $\lambda x.(C(x) \land P(\varepsilon(x)))$ in the conjunction of the sentences (a), (b) and (c).

Let $A(P;Z)$ be a sentence of L_E, P a predicate constant and Z a tuple of the other predicate constants which occur in A. The connotative circumscription of P in A with respect to F is a sentence

$$
\begin{aligned}
& A(P;Z)^{*D;F} \wedge A_N{}^{*N;G} \wedge \varepsilon\text{-axiom}(C;D;\varepsilon;F) \\
& \wedge \forall p,z,c,d,\varepsilon',F'.(\ A(p;z)^{*d;F'} \wedge \varepsilon\text{-axiom}(c,d,\varepsilon',F') \\
& \qquad\qquad \wedge \forall x.(\ c(x) \wedge p(\varepsilon'(x)) \supset C(x) \wedge P(\varepsilon(x))\ ) \\
& \qquad\qquad \supset \forall x.(\ c(x) \wedge p(\varepsilon'(x)) \equiv C(x) \wedge P(\varepsilon(x))\ )\ ), \quad (13)
\end{aligned}
$$

which is denoted by C-Circ(A;P;F).

This sentence can be easily generalized from a predicate P to a tuple of predicates by the similar way in circumscription.

In the next section, we apply this sentence and see how it works.

3. Examples

Example 1 (continued).

We connotatively circumscribe Ab with respect to anonym and tweety, that is, P is Ab and F is the tuple of anonym and tweety in C-Circ(A;P;F).

$$
\begin{aligned}
& A^{*D;F} \equiv \exists x.D(x) \wedge A^D \wedge Func^D(anonym,tweety), \qquad (14) \\
& A^D \quad \equiv \forall x.(\ D(x) \wedge Knocks(x) \wedge \neg Ab(x) \supset Man(x)\ ) \\
& \qquad \wedge Knocks(anonym) \wedge Ab(tweety) \wedge \neg Man(tweety) \qquad (15) \\
& Func^D(anonym,tweety) \\
& \qquad \equiv D(anonym) \wedge D(tweety) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
& \varepsilon\text{-axiom}(C;D;\varepsilon;F) \\
& \qquad \equiv \forall x.(C(x) \supset D(\varepsilon(x))) \\
& \qquad\quad \wedge anonym = \varepsilon(\text{`}anonym) \wedge C(\text{`}anonym) \\
& \qquad\quad \wedge tweety = \varepsilon(\text{`}tweety) \wedge C(\text{`}tweety) \qquad (17)
\end{aligned}
$$

Notice that 'anonym and 'tweety represent natural numbers distinctly, for instance, 'anonym = 0 and 'tweety = 1.

In C-Circ(A;Ab;anonym,tweety), if we substitute

$$
\begin{aligned}
& \varepsilon' = \lambda x.x, \\
& c = d = \lambda x.(x = \text{`}anonym \vee x = \text{`}tweety), \\
& anonym' = \text{`}anonym, \\
& tweety' = \text{`}tweety, \\
& knocks = \lambda x.(x = \text{`}anonym), \\
& ab = \lambda x.(x = \text{`}tweety), \\
& man = \lambda x.(x \neq tweety),
\end{aligned}
$$

it yields

$$
\begin{aligned}
& \forall x.((x = \text{`}anonym \vee x = \text{`}tweety) \wedge x = \text{`}tweety \supset C(x) \wedge Ab(\varepsilon(x))) \\
& \supset \forall x.((x = \text{`}anonym \vee x = \text{`}tweety) \wedge x = \text{`}tweety \equiv C(x) \wedge Ab(\varepsilon(x))). \\
& \qquad (18)
\end{aligned}
$$

The left side of this implication is seen to be true, so this gives

$$\forall x.(x = \text{'tweety} \equiv C(x) \wedge Ab(\varepsilon(x))). \quad (19)$$

Then if we substitute 'anonym for x in (19), we can obtain

$$\neg Ab(\varepsilon(\text{'anonym})) \quad (20)$$

that is, from ε-axiom, (17), (15) and (16)

$$\neg Ab(anonym) \wedge Man(anonym) \quad (21)$$

follows.

(19) says that the only object named Tweety abnormal. Anonym has another name (function symbol), so (21) holds. Notice that (21) is our intended result but usual circumscription can never yield it. Moreover, (19) say nothing about objects with no name, so abnormal things may still exist somewhere. That is, this shows that connotative treatment is a hopeful method for the problems of absence of abnormal things and limitation on equality.

Example 2. The unique name assumption.

Connotative treatment is also successful in formalizing the unique name assumption. We connotatively circumscribe equality, =.

Example 2.a.

Let A be empty and F be a tuple of finite function symbol. Then $A^{*D;F}$ is the conjunction of a sentence $\exists x.D(x)$ and finite sentences $\forall x_1,\ldots,x_n.(\ \Phi(x_1) \wedge \ldots \wedge \Phi(x_n) \supset \Phi(f(x_1,\ldots,x_n))\ )$ for each n-ary functions in F. And ε-axiom(C;D;ε;F) is the conjunction of a sentence $\forall x.(C(x) \supset D(\varepsilon(x)))$ and sentences $\forall x_1,\ldots,x_n.(\ C(x_1) \wedge \ldots \wedge C(x_n) \supset f(\varepsilon(x_1),\ldots,\varepsilon(x_n)) = \varepsilon(\text{'}f(x_1,\ldots,x_n)) \wedge C(\text{'}f(x_1,\ldots,x_n))\ )$ for each n-ary functions in F. Substituting C for c, C for d, f for 'f and λx.x for ε', $A^{*d;F'}$ follows from ε-axiom(C;D;ε;F), and ε-axiom(c;d;ε';F') ≡ $A^{*d;F'}$ holds. Therefore, C-Circ(A; =;F) yields

$$\forall x,y.(C(x) \wedge C(y) \wedge x = y \supset C(x) \wedge C(y) \wedge \varepsilon(x) = \varepsilon(y)) \\ \supset \forall x,y(C(x) \wedge C(y) \wedge x = y \equiv C(x) \wedge C(y) \wedge \varepsilon(x) = \varepsilon(y)) \quad (22)$$

Because the left side of (22) holds, we can obtain

$$\forall x,y(C(x) \wedge C(y) \supset (x = y \equiv \varepsilon(x) = \varepsilon(y))). \quad (23)$$

(23) says that distinct names denotes distinct objects. This is the narrow definition of the unique name assumption itself.

Notice that though there exist infinite names (i.e. natural numbers) this formalization is finitely axiomatizable if a function set, F, is finite,.

Example 2.b.

Let A be a sentence

$$\forall x.(x=e_0 \vee x=e_1) \wedge P(e_0) \wedge a_0 \neq a_1, \qquad (24)$$

and F be e_0,e_1,a_0,a_1. C-Circ(A;=;e_0,e_1,a_0,a_1) yields

$$P(a_0) \supset \forall x.(\,(x='e_0 \vee x='a_0) \equiv C(x) \wedge P(\varepsilon(x))\,). \qquad (25)$$

From (23) and ε-axiom,

$$P(a_0) \supset P(e_0) \wedge \neg P(e_1) \wedge P(a_0) \wedge \neg P(a_1) \qquad (26)$$

follows. Similarly,

$$P(a_1) \supset P(e_0) \wedge \neg P(e_1) \wedge \neg P(a_0) \wedge P(a_1) \qquad (26)$$

holds. $P(a_0) \vee P(a_1)$ holds, therefore

$$P(e_0) \wedge \neg P(e_1) \wedge P(a_0) \wedge \neg P(a_1) \vee P(e_0) \wedge \neg P(e_1) \wedge \neg P(a_0) \wedge P(a_1) \qquad (27)$$

This shows the result of minimization of equality.

4. Conclusion and Future Work

We proposed connotative circumscription which minimize the set of names which denote objects satisfying a certain predicate. It provides a solution for the three problem; absence of abnormal things, limitation of circumscription on equality and formalizing the unique name assumption. In our future work, we attempt to clarify properties of this connotative circumscription, especially, we are interested in consistency of this sentence. Circumscription cannot preserve consistency generally, however, how about this version?

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