ICOT Technical Memorandum: TM-0635

TM-0635

A Study of a Weak Point of Circumscription on Equality

by
J. Arima

December, 1988

ICOT

Mita Kokusai Bldg. 21F
4-28 Mita 1-Chome
Minato-ku Tokyo 108 Japan

(03) 456-3191~5
Telex ICOT J32964

Institute for New Generation Computer Technology

A Study of a Weak Point of Circumscription on Equality

(EXTENDED ABSTRACT)

Jun ARIMA

ICOT Research Center

4-28 Mita 1-chome, Minato-ku, Tokyo, 108, Japan

Phone: +81 3 456 4365, C.Mail Address: arima%icot.jp@relay.cs.net

Topic: Nonmonotonic Reasoning, Knowledge Representation Formalism
Keyword: Circumscription

Abstract

This paper points out a problem, called the anonym problem, that circumscription is weak in proving inequality, and clarifies a limitation of circumscription in proving (in)equality. It also proposes a solution to the problem.

1. Introduction

We, humans, are often faced with a lack of available information in solving problems. In some cases, we infer from imperfect knowledge by complementing it with our common sense knowledge, such as " a bird normally flies", without violating consistency. To capture such reasoning formally, McCarthy proposes a form called circumscription [7,8]. However, unfortunately, circumscription is weak in proving inequality and its weakness yields only unsatisfactory results under circumstances where lack of information regarding inequality is essential. We show this with an instance, called the anonym problem. Then we clarify a limitation of circumscription in yielding new facts regarding (in)equality. We also proposes a solution to the problem is proposed.

2. Anonym Problem

First, we must review circumscription. Circumscription is well known to have various versions. In this paper, we consider the parallel circumscription [4] proposed by Lifschitz, because parallel circumscription is a sufficiently general and powerful version in that most of the other versions, for example, predicate circumscription [7], formula circumscription [8] or prioritized circumscription [4,8], can be expressed using this formulation [4,9]. In this paper, we refer to this parallel circumscription simply as "circumscription".

We basically follow Lifschitz's definitions and notation in [4]. For any first order sentence, A(P,Z), where P is a tuple of distinct predicate constants and Z a tuple of function and/or predicate constants disjoint with P, the circumscription of P in A with variable Z is the sentence

$$A(P,Z) \wedge \neg \exists p,z.(A(p,z) \wedge p < P), \qquad (1)$$

denoted by Circum(A;P;Z). Here, p and z are tuples of variables, and $p<P$ expresses that the extension of each member of p is a subset of the extension of the corresponding member of P, and at least one of them is a proper subset. When we need to minimize wff E(x), we especially allow it to be used in the following way,

$$\mathrm{Circum}(A \wedge \forall x.(P_0(x) \equiv E(x));P_0;Z), \qquad (2)$$

where P_0 is a new predicate which does not occur in A. (This is essentially equivalent to formula circumscription and variable circumscription [9].)

Now, let us consider the following situation.

Situation 1:

A man in a living room hears someone knocking on the front door. He knows that if someone knocks it is normally a man...

Under Situation 1, we expect that he would think that the anonym is a man. In this case, circumscription works as our expectation.

His knowledge can be written as follows:

$$A = \mathrm{Knocks(Anonym)} \wedge \forall x.(\mathrm{Knocks}(x) \wedge \neg \mathrm{Ab}(x) \supset \mathrm{Man}(x)).$$

We circumscribe predicate Ab, allowing predicate Man to vary, that is,Circum(A; Ab; Man) yields

Man(Anonym).

Situation 2 (continuing from Situation 1):

...He opens the door, and finds not a man but a woodpecker, Tweety (Tweety is abnormal). Tweety flies away... He goes back to the living room and he stays there. He hears someone knocking...

Under Situation 2, although he knows of the existence of an abnormal being, we still expect that he would think that the anonym (Anonym') is a man, because he still knows that a knocker is normally a man. Now we write

$A' = Knocks(Anonym') \wedge Ab(Anonym) \wedge \forall x.(Knocks(x) \wedge \neg Ab(x) \supset Man(x))$

(to simplify and clarify the point, Knocks(Anonym) is abandoned).

In this case, does circumscription work as our expectation? By only circumscribing Ab with variable Man, we cannot obtain the expected result. We can obtain only

$$Anonym' \neq Anonym \supset Man(Anonym').$$

This implies that he cannot conclude that this Anonym' is a man as long as he does not know that Anonym' is not the latter Anonym. Staying in his living room, he can never infer that this anonym would be a man; moreover, even if he sees what this anonym is, it will imply knowing whether he is a man. It seems more absurd for him to remember all abnormal things so exactly that he can distinguish them from whomever he meets.

Circumscription seems to be weak in proving inequalities as we have shown above. We call such difficulties of circumscription that arise in proving with respect to equalities an "anonym problem". Of course, before we call it a "problem", some problems still have to be clarified. Is it really impossible to deduce the above inequality (Anonym'$\neq$ Anonym) from any instances of circumscription? (That is, do there exist some minimized predicates (or formulas) and constants (predicates or functions) allowed to vary, with which the inequality is deduced?) In general, what inequality can be deduced from circumscription, and what equality? In the next section, we will clarify these problems.

3. Limitation of Circumscription on Equality

Our interest is knowing the ability of circumscription in yielding new facts regarding (in)equality which are not yielded without circumscription. Before exploring the ability generally, we solve the following question: Can circumscription yield no new (in)equality at all? The answer is "No, it cannot". We can show counterexamples to the question, which instance that circumscription yields new facts regarding equality in some cases.

Example 1 (on equality): $A1 \equiv Ab(a) \wedge Ab(b)$. Circum(A1;Ab;b) yields $\forall x.(Ab(x) \equiv x=a)$. From this, we can obtain a new fact regarding equality

$a = b$.

Example 2 (on inequality): $A2 \equiv Q(a) \wedge \neg Q(b) \wedge R(c)$. Circum($A2 \wedge \forall x.(P_0(x) \equiv \neg(R(x) \equiv Q(x)))$; P_0; R, c) yields $\forall x. \neg P_0(x)$,

therefore, $\forall x.(R(x) \equiv Q(x))$ and then Q(c) are obtained. Therefore,

$b \neq c$.

These examples seem to imply the possibilities that circumscription yields any new facts regarding equality that we want; however, it turns out to be negative. Here we show this.

We discuss this problem from a model-theoretic standpoint. We use the following notation.

A as a first order sentence, Mod(A) as the class of models of A,
[M] as the domain of model structure M, M[P] as the extension of P in M,
|S| as the cardinal number of a set S, ω as the least infinite ordinal number, and
(P;Z)min-Mod(A) as the class of (P;Z)-minimal models of A.

To start with, we need to recall the model-theoretic meaning of circumscription. A pre-order on the class of structures "$\leq^{P;Z}$" was introduced [4]. For any structures, M_1 and M_2, we write $M_1 \leq^{P;Z} M_2$ if 1) $[M_1] = [M_2]$, 2) $M_1[K] = M_2[K]$ for every constant K not in P,Z, and 3) $M_1[P_i] \subseteq M_2[P_i]$ for every P_i in P. We say that a structure is a *(P;Z)-minimal* *model of A* if the structure is a minimal model of A with respect to $\leq^{P;Z}$. Then a model of Circum(A:P;Z) is equivalent to a (P;Z)-minimal model of A [5].

The following theorem holds.

Theorem:

For any (A, P, Z), for any $M_0 \in Mod(A)$ such that $|[M_0]| \leqq |[M]|$ for all $M \in Mod(A)$, and for any W such that

$$Circum(A; P; Z) \vdash W \quad (\text{or } Circum(A \wedge \forall x.(P_0(x) \equiv E(x)); P; Z) \vdash W),$$

if there exists a model of A whose domain consists of finite objects, then

$$|[M_0]| = |[N_0]|$$

for any $N_0 \in Mod(A \wedge W)$ such that $|[N_0]| \leqq |[N]|$ for all $N \in Mod(A \wedge W)$. ■

This theorem states that if there exists a model whose domain consists of finite individuals, any circumscription expressible by (1) or (2) cannot yield a new fact that varies the least cardinal number of models. This essentially implies that circumscription is weak in proving *inequality* which causes an increase in the least cardinality. Up to the present, the fact which shows the weakness of circumscription in proving *equality* has not been found.

Example 1 (continued): A structure, M, is a model of A1 such that M[Ab] = {M[a]} = {M[b]}. Therefore, the least cardinal number of A1 models is 1. Also, the least cardinal number of Circum(A1; Ab; b) models is 1.

Example 2 (continued): Both the least cardinal number of A2 models and that of $Circum(A2 \wedge \forall x.(P_0(x) \equiv \neg(R(x) \equiv \neg Q(x))); P_0; R, c)$ are 2.

Example 3 (continued from Situation 2): The least cardinal number of A' is 1 because there exists a model of A' whose domain is a singleton. If circumscription yields (Anonym' ≠ Anonym), then the least cardinal number of its models must be greater than (and not equal to) 1, at least. Therefore, circumscription never states this.

Etherington et al [2,3] showed that circumscription without variables is weak in proving equality. We have shown more general results on it. He also pointed out that circumscription without variable terms cannot conjecture "a $\neq$ b" in the absence of any knowledge, while the default theory can [3]. Our result shows that circumscription even with variables can not yield such inequalities (if it can, the least cardinal number changes from 1 to 2). This results in the fact that circumscription cannot subsume default logic.

4. A Solution of the Anonym Problem

We show a solution of the anonym problem by proposing a new version of circumscription. The key of our treatment is to interpret a sentence, A, in the infinite domain which consists of natural number and to deal with connotative objects which occur in the language rather than denotative objects which exist in the domain. The new version enables us to change the interpretation in *de dicto - de re* of an object so that it might not satisfy a certain property, even allowing the domain of A to vary. Although this notion expresses a local minimality condition which expresses the impossibility of changing the value of a predicate from true to false at one "connotative" point, notice that it does not necessarily imply the minimality of extension of a predicate, while the notion of (pointwise) circumscription [6] does imply this minimality. Because the change only occur in the *de dicto - de re* relation of an object and does not occur in the interpretation of predicate. (This connotative treatment provides some advantages other than those related to the problem. These advantages will also be reported).

To deal with the domain of a discourse, we use a technique which is proposed in minimizing the domain [1, 7].

N stands for:

$N(0) \land \forall x.(N(x) \supset N(s(x))) \land \forall x.(0 \neq s(x)) \land \forall x,y.(s(x)=s(y) \supset x=y)$,

where N, 0 and s are a unary predicate symbol, 0-ary function symbol and unary function symbol, respectively, all of which do not occur in A. Let *Axiom*(D) be the conjunction of

$\forall x_1,\ldots,x_n.(D(x_1)\wedge\ldots\wedge D(x_n) \supset D(f(_1,\ldots,x_n)))$ for each n-ary function symbol f. Let Ψ be a sentence, then let $\{\Psi\}^D$ be the result of replacing each universal quantifier prefixed to an object variable, "$\forall x.\cdots$", in Ψ with "$\forall x.(D(x) \supset \cdots)$" and each existential quantifier prefixed to an object variable, "$\exists x. \cdots$ " with "$\exists x.(D(x) \wedge \cdots)$". E(x,Z) be a wff. Then the *connotatively pointwise circumscription (cp-circumscription)* of E with respect to t in A with variable Z is

$$N \wedge Axiom(D) \wedge \{A(t,Z)\}^D \wedge \neg \exists d.(\{\exists x,z.(A(x,z) \wedge \neg E(x,z))\}^d \wedge \{E(t,Z)\}^D), \qquad (3)$$

denoted by Cp-Circum(A; E; t; Z).

This formula requires a model of A which cannot be transformed into another model of A by changing an object indicated by name 't' from one satisfying E to another not satisfying E, even allowing Z and the domain to vary.

This new version partly realizes the unique name assumption [10]. Consider the following examples.

Example 4 (reviewing the Etherington's example): Let "true" be a predicate which is valued *true*. Consider Cp-Circum(true; $\lambda x,y.(x=y)$; a, b), that is,

$$N \wedge (D(a) \wedge D(b)) \wedge \{true\}^D \wedge \neg \exists d.(\{\exists x,y.(true(x,y) \wedge x \neq y)\}^d \wedge \{a=b\}^D),$$

which simplifies to

$$N \wedge (D(a) \wedge D(b)) \wedge \forall d.(\exists x,y.(d(x) \wedge d(y) \wedge x \neq y)) \supset \neg \{a=b\}^D).$$

If we now substitute true for d, it yields

$$\exists x,y.(x \neq y) \supset \neg\{a=b\}^D.$$

Of course, the precondition follows from N, and, since $(D(a) \wedge D(b))$ holds,

$$a \neq b.$$

Example 3 (continued): Cp-Circum(A'; Ab; Anonym'; Ab, Knocks, Man, Anonym) yields $\neg$Ab(Anonym').

5. Conclusion

We have discussed the fact that circumscription is weak to prove inequality, and proposed a new version of circumscription which increases the power in dealing with each connotative object. The new version partly realizes the unique name assumption.

ACKNOWLEDGMENTS

I wish to express my gratitude to Dr. Kazuhiro Fuchi, Director of the ICOT Research Center, who provided me with the opportunity to pursue this research.

References

[1]: Davis,M.: The mathematics of non-monotonic reasoning, *Artificial Intelligence* 13 (1980) 73-80.

[2]: Etherington,D.,Mercer,R. and Reiter,R.: On the adequacy of predicate circumscription for closed-world reasoning, Technical report 84-5, Dept. of Computer Science, Univ. of British Columbia (1984).

[3] Etherington,D.: Relating Default Logic and Circumscription, *Proc. of Tenth International Joint Conference on Artificial Intelligence*, Milan, Italy (1987) 489-494.

[4]: Lifschitz,V.: Computing circumscription, *Proc. of Ninth International Joint Conference on Artificial Intelligence*, Los Angeles, CA (1985) 121-127.

[5] Lifschitz,V.: On the Satisfiability of Circumscription, *Artificial Intelligence* 28 (1986) 17-27.

[6]: Lifschitz,V.: Pointwise Circumscription, *Proc. of AAAI-86* (1986) 406-410.

[7]: McCarthy,J.: Circumscription - a form of non-monotonic reasoning, *Artificial Intelligence* 13 (1980) 27-39.

[8]: McCarthy,J.: Application of circumscription to formalizing common-sense knowledge, *Artificial Intelligence* 28 (1986) 89-116.

[9]: Perlis,D. and Minker,J.: Completeness Results for Circumscription, *Artificial Intelligence* 28 (1986) 29-42.

[10]: Reiter,R.: Equality and domain closure in first-order data bases, *JACM* 27 (1980) 235-249.