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Ascription: An Approach to Formalization of Non-monotonic Reasoning in the Wider Sense

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Abstract

The introduction of new axioms can invalidate old theorems obtained by "non-monotonic" reasoning [8]. We can also consider induction and analogy, not only common sense reasoning, to be non-monotonic reasoning in that such reasoning processes have non-monotonicity in themselves.

Here, a logical framework is proposed that draws hypotheses from which properties of unknown facts are deduced by relativizing and generalizing already acquired knowledge. It is called *ascription*. Ascription is an approach to the uniforming formalization of such conjectural reasoning. The basic idea is this: when all the demonstrated positive instances of a certain property K are also shown to have a property Ψ, and similarly all the demonstrated negative instances of K are shown not to have the property Ψ, the property K may be equivalent to Ψ. It is realized in this logical framework using predicate substitution. The preservation of consistency by ascription is also discussed. This result can be applied to similar logical frame work such as circumscription[6,7].

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1.Introduction

Computer systems with capabilities of deductive inference will release man from the troublesome tasks of procedural programming and be able to solve problems which are given only declaratively. But deductive inference is deduction of properties of individuals from given general knowledge, so it cannot provide effective consequences about facts that are unexpected and not included in general knowledge. This means that deductive inference cannot make a significant contribution to solving our software crisis. We should remember unexpected facts always exist and deductive inference is helpless with respect to them. One promising approach is reasoning by relativizing and generalizing already acquired knowledge so that it can be applied to unexpected circumstances.

Work related to this kind of reasoning was done by John McCarthy et al. *Circumscription* [6,7] is a form of such conjectural reasoning as is done by humans and based on the closed-world assumption. This work is important and interesting, but it seems that it can explain only a small part of human flexible reasoning and that there still remain some very important aspects which we should not ignore. These are analogy, induction and other reasoning processes which strongly relativize and generalize knowledge. Such reasoning is closely related to human learning capabilities. We have studied reasoning from this point of view and propose a logical framework, called *ascription*, which is a form of such conjectural reasoning. Intuitively, ascription represents the flexible notion that the interpretation of a certain property K lies between two extremes; one, similar to predicate circumscription, that the only demonstrated positive instances of K are all instances satisfying K; and the other, that all but the demonstrated negative instances satisfy K. More precisely we will show this as follows.

Ascription is based on the following notion: *if all the entities that can be shown to have a property K by reasoning from already acquired knowledge Γ can also be shown to have a property Ψ, and all the entities that can be shown not to have the property K can also be shown not to have the property Ψ, the property K may be equivalent to Ψ.* Namely when all the demonstrated positive instances of K are positive instances of Ψ, and similarly all the demonstrated negative instances of K are negative instances of Ψ, we can assume the equivalence of K and Ψ.

2. Ascription schema

In this paper we write **t** instead of a tuple of finite terms for brevity. For example, a formula $A(\mathbf{x})$ stands for $A(x_1,\cdots,x_j)$ and the quantifier $\forall\mathbf{x}.$ stands for $\forall x_1.\cdots.\forall x_k$. And by a finite set of formulas $\{ F_1, F_2, \cdots, F_m \}$, we mean a formula $F_1{}^\circ \wedge F_2{}^\circ \wedge \cdots \wedge F_m{}^\circ$, where $F_i{}^\circ$ $(i=1,\cdots,m)$ is a closed formula obtained from F_i by prefixing $\forall$ with respect to all of the free variables in F_i.

By n-ary predicate we mean an expression $\lambda\mathbf{x}.(A(\mathbf{x}))$, where $\mathbf{x}$ is a tuple of n variables and $A(\mathbf{x})$ is a formula in which $\mathbf{x}$ occurs free and no other variables occur free. That is, a predicate is obtained from a formula by λ-abstracting all of the free variables in it.

Let K be a tuple of distinct predicate symbols, K1,⋯,Kn, and Ψ a tuple of predicates, Ψ1,⋯,Ψn, where Ki and Ψi have the same arity. [Ψ/K] means a substitution, representing [Ψ1/K1,⋯,Ψn/Kn] and usually abbreviated [Ψ]. We write $A(\mathbf{x})[\Psi/K]$ for the result of replacing simultaneously each occurrence Ki in $A(\mathbf{x})$ by Ψi. And similarly, $[\lambda\mathbf{x}.(\Psi(\mathbf{x}))]$ stands for $[\lambda\mathbf{x}_1.(\Psi_1(\mathbf{x}_1)),\cdots,\lambda\mathbf{x}_n.(\Psi_n(\mathbf{x}_n))]$, and $\forall\mathbf{x}.(\ K(\mathbf{x}) \equiv \Psi(\mathbf{x})\ )$ stands for $\forall\mathbf{x}_1.(\ K_1(\mathbf{x}_1) \equiv \Psi_1(\mathbf{x}_1)\ )\wedge\cdots\wedge\forall\mathbf{x}_n.(\ K_n(\mathbf{x}_n) \equiv \Psi_n(\mathbf{x}_n)\ )$ (where $\forall\mathbf{x}_i.(\ K_i(\mathbf{x}_i) \equiv \Psi_i(\mathbf{x}_i)\ )$ means $\forall\mathbf{x}_i.(\ K_i(\mathbf{x}_i) \supset \Psi_i(\mathbf{x}_i)\ ) \wedge \forall\mathbf{x}_i.(\ K_i(\mathbf{x}_i) \subset \Psi_i(\mathbf{x}_i)\ )$).

<u>Definition</u> [*Ascription schema*].

Let K be a tuple of distinct predicate symbols. And let Γ be a set of formulas of first order logic containing all predicates in K. The *ascription of* K *to* Ψ *in* Γ[K] is the schema

$$\Gamma[\lambda\mathbf{x}.(K(\mathbf{x})\triangle\Psi(\mathbf{x}))] \ \wedge\ \Gamma[\lambda\mathbf{x}.(K(\mathbf{x})\triangledown\Psi(\mathbf{x}))] \ \supset\ \forall\mathbf{x}.(\ K(\mathbf{x}) \equiv \Psi(\mathbf{x})\ ). \quad \cdots\cdot \quad (1)$$

Here △,▽ represent respectively ∧,∨ or ∨,∧ in each corresponding predicate. And Ψ is a tuple of predicates which have the same arity as the corresponding predicates in K. We call the formula on the left side of this schema the *ascribable condition* writing As(Γ,K~Ψ).

$\Gamma[\cdots,\lambda\mathbf{x}i.(Ki(\mathbf{x}i)\wedge\Psi i(\mathbf{x}i)),\cdots]$ expresses the assumption that all the tuples of entities that can be shown to have a certain property Ki by reasoning from certain facts Γ can also be shown to have a certain property Ψi. $\Gamma[\cdots,\lambda\mathbf{xi}.(Ki(\mathbf{xi})\vee\Psi i(\mathbf{xi})),\cdots]$ is, as far as Ki is concerned, equivalent to the result of replacing $\neg Ki$ by $\lambda\mathbf{xi}.(\neg Ki(\mathbf{xi})\wedge\neg\Psi i(\mathbf{xi}))$. Namely $\Gamma[\cdots,\lambda\mathbf{xi}.(Ki(\mathbf{xi})\vee\Psi i(\mathbf{xi})),\cdots]$ expresses the assumption that all the tuples of entities that can be shown not to have a property Ki, can also be shown not to have a certain property Ψi. When we can assume that both $\Gamma[\cdots,\lambda\mathbf{xi}.(Ki(\mathbf{xi})\wedge\Psi i(\mathbf{xi})),\cdots]$ and $\Gamma[\cdots,\lambda\mathbf{xi}.(Ki(\mathbf{xi})\vee\Psi i(\mathbf{xi})),\cdots]$ are true, (1) lets us conclude the formula on the right side, namely that Ki is equivalent to Ψi.

It is not allowed that, regarding (1) as a second order formula quantified by $\forall\Psi$, we add it to Γ. We must use ascription in the following way.

When the formula p follows from a set of formulas Γ by a complete deduction system of first order logic, we write $\Gamma \vdash p$. Let $\Gamma_{h-1}\{K^h\sim\Psi^h\}$ be $\Gamma_{h-1} \cup \{ As(\Gamma_{h-1},K^h\sim\Psi^h) \supset \forall\mathbf{x}.(\ K^h(\mathbf{x}) \equiv \Psi^h(\mathbf{x})\ )\}$ ($h = 1,2,\cdots$) and Γ_0 be Γ. Let Γ_h be $\Gamma_{h-1}\{K^h\sim\Psi^h\}$, written $\Gamma\{K^1\sim\Psi^1;\cdots;K^h\sim\Psi^h\}$. If a finite number n exists such that $\Gamma_n \vdash p$, we write $\Gamma \mid\sim_{\{K^1\sim\Psi^1;\cdots;K^n\sim\Psi^n\}} p$ and usually abbreviate this as $\Gamma \mid\sim p$.

Example 1. Let Γ be a set of relations among 'animate', 'mammal' and 'human' and some instances. Suppose we want to know what 'homoiothermal' is. Ascription develops the candidates for a concept equivalent to 'homoiothermal'. Γ may be

$$
\begin{aligned}
\Gamma \equiv\ & \Gamma[\mathrm{Homoiothermal}] \\
\equiv \{\ & \forall x.(\mathrm{Human}(x) \supset \mathrm{Mammal}(x)) , \\
& \forall x.(\mathrm{Mammal}(x) \supset \mathrm{Animate}(x)) , \\
& \exists x.(\neg \mathrm{Human}(x) \wedge \mathrm{Mammal}(x)) , \\
& \exists x.(\neg \mathrm{Mammal}(x) \wedge \mathrm{Animate}(x)) , \\
& \exists x.(\mathrm{Human}(x) \wedge \mathrm{Homoiothermal}(x)) , \\
& \exists x.(\mathrm{Animate}(x) \wedge \neg \mathrm{Homoiothermal}(x)) \}.
\end{aligned}
$$

Let us consider 'mammal' as a candidate property equivalent to 'homoiothermal'. First we check whether 'mammal' can be equivalent to 'homoiothermal' or not, namely check the ascription condition As(Γ,Homoiothermal~Mammal).

$$
\begin{aligned}
& \Gamma[\lambda x.(\mathrm{Homoiothermal}(x) \wedge \mathrm{Mammal}(x))] \equiv \\
\{\ \ & \forall x.(\mathrm{Human}(x) \supset \mathrm{Mammal}(x)) , \\
& \forall x.(\mathrm{Mammal}(x) \supset \mathrm{Animate}(x)) , \\
& \exists x.(\neg \mathrm{Human}(x) \wedge \mathrm{Mammal}(x)) , \\
& \exists x.(\neg \mathrm{Mammal}(x) \wedge \mathrm{Animate}(x)) , \\
& \exists x.(\mathrm{Human}(x) \\
& \quad \wedge (\mathrm{Homoiothermal}(x) \wedge \mathrm{Mammal}(x))) ,
\end{aligned}
$$

$\exists x.(Animate(x)$
$\wedge \neg(Homoiothermal(x) \wedge Mammal(x)))\ \}$

$\Gamma[\lambda x.(Homoiothermal(x) \vee Mammal(x))] =$
$\{ \cdots, \exists x.(Human(x)$
$\wedge (Homoiothermal(x) \vee Mammal(x)))\ ,$
$\exists x.(Animate(x)$
$\wedge \neg(Homoiothermal(x) \vee Mammal(x)))\ \}$

Clearly $\Gamma \vdash \Gamma[\lambda x.(Homoiothermal(x) \wedge Mammal(x))] \wedge \Gamma[\lambda x.(Homoiothermal(x) \vee Mammal(x))]$, so

$\Gamma \cup \{ As(\Gamma, Homoiothermal \sim Mammal)$
$\supset \forall x.(Homoiothermal(x) \equiv Mammal(x))\ \}$
$\vdash \forall x.(Homoiothermal(x) \equiv Mammal(x)).$

This shows that 'homoiothermal' may be 'mammal'. Therefore, we get

$\Gamma\ |\!\sim\ \ \forall x.(Homoiothermal(x) \equiv Mammal(x))$
$\wedge\ \forall x.(Human(x) \supset Homoiothermal(x))$
$\wedge\ \forall x.(Homoiothermal(x) \supset Animate(x)).$

Of course we can also regard 'human' as another candidate. But what is important here is the fact that reasoning by ascription can derive a "medium" interpretation of K.

Notice that these inferences cannot be derived by circumscription [6] (even *formula* circumscription [7]). Because circumscription is based on partial models which are only minimal with respect to some properties.

Now if we add the axiom

$\exists x.(\neg Homoiothermal(x) \wedge Mammal(x))$

then $\Gamma'[\lambda x.(Homoiothermal(x) \vee Mammal(x))]$ is inconsistent, so $\forall x.(Homoiothermal(x) \equiv Mammal(x))$ is not a theorem of the extended theory Γ', where $\Gamma' = \Gamma \cup \{ \exists x.(\neg Homoiothermal(x) \wedge Mammal(x))\ \}$. This shows that reasoning by ascription is non-monotonic.

3.Model theory of ascription

For the model theory of ascription, we introduce *most* Ψ*-tending model.*

Definition [Ψ*-tending model* in K].

Let M(Γ) and N(Γ) be models of the sentence Γ. We say M is a more Ψ*-tending model* than N in K, writing $M \geqq_{K\sim\Psi} N$, if M and N have the same domain, and if all other predicate symbols not in K have the same extensions in M and N, but the extension of $\lambda\mathbf{x}.(K(\mathbf{x})\wedge\Psi(\mathbf{x}))$ in M includes its extension in N and the extension of $\lambda\mathbf{x}.(\neg K(\mathbf{x})\wedge\neg\Psi(\mathbf{x}))$ in M includes its extension in N.

Definition [*most* Ψ*-tending model* in K].

A model M of Γ is called *most* Ψ*-tending* in K iff $M' \geqq_{K\sim\Psi} M$ only if $M' =_{K\sim\Psi} M$ (where by " $M' =_{K\sim\Psi} M$ " we mean " $M' \geqq_{K\sim\Psi} M$ and $M' \leqq_{K\sim\Psi} M$ ").

4.On consistency of ascription

In this section we propose conditions which are sufficient for ascription to preserve consistency. That is, these conditions guarantee that a consistent Γ cannot contradict the result derived by ascription. Moreover, we can also show that under these conditions any instance of ascription is true in all the most Ψ-tending models. Before going any further, we have to consider substitution for predicate. We start by introducing the concept of free substitution.

By a *free* substitution we mean a substitution which is free in the sense of [2]. Namely a free substitution satisfies the following two conditions. (A1) In replacing predicate symbols K in a formula F by some predicate Ψ, any variable in each terms **u** attaching K in F must be free in Ψ(**u**); and (A2) any free variable (not bound by quantifiers and not λ-abstracted) in Ψ must remain free in F[Ψ/K]. But notice in this paper we regard any predicate as closed λ-expression, so (A2) is always satisfied. Moreover, by renaming adequate variables in Ψ we can always ensure that (A1) is satisfied. So, in this paper, substitution represented by a pair of brackets [⋯] is always free.

Theorem 1 Let Γ be a set of formulas of first order logic and p be a formula of first order logic. If Γ is consistent and some free substitution Θ exists such that $\Gamma \vdash \Gamma\Theta \wedge p\Theta$, then $\Gamma \cup \{p\}$ is consistent.

Proof. Let Θ be a free substitution such that $\Gamma \vdash \Gamma\Theta \wedge p\Theta$. Now we assume that $\Gamma \cup \{p\} \vdash \Box$ (representing 'false'). Namely $\Gamma \vdash \neg p$. Using Kleene's theorem on substitution for predicate letters ([2], we refer to this theorem as *Kleene's theorem* below), $\Gamma\Theta \vdash \neg p\Theta$

follows. Here $\Gamma \vdash \Gamma\Theta$, so $\Gamma \vdash \neg p\Theta$. This result contradicts the assumption that $\Gamma \vdash p\Theta$.

If S is a set of substitutions, S* denotes the set of all the substitutions which are represented by finite sequence of elements of S.

Definition [*well-representative form*].

Ψ is a *well-representative form* of K if for some finite sequence of free substitutions $\Theta_K \in \{ [\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))], [\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))] \}^*$ exists such that $\Gamma \vdash \forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))\Theta_K$.

By well-representative condition we mean that Ψ is a *well-representative form* of K.

Now we introduce some characteristic types of well-representative form.

Prop. 1 [type 1].

If $\Gamma \vdash \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))]) \wedge \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))])$ then Ψ is a well-representative form of K.

Proof. $\Gamma \vdash \forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))] \cdot [\lambda \mathbf{x}.(K(\mathbf{x}) \vee \Psi(\mathbf{x}))]$.

The condition of type 1 means that Ψ is not affected by change of interpretation of K.

Prop. 1.1.

$\Gamma \vdash \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))]) \wedge \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))])$ if Ψ can be transformed into an expression of the form $\lambda \mathbf{x}.(K(\mathbf{x}) \wedge G(\mathbf{x}) \vee H(\mathbf{x}))$, where G, H are tuples of predicates in which no predicate symbols in K occur.

Proof. By predicate calculus.

Notice that each of the forms, $H(\mathbf{x})$, $\lambda \mathbf{x}.(K(\mathbf{x}) \vee H(\mathbf{x}))$ and $\lambda \mathbf{x}.(K(\mathbf{x}) \wedge G(\mathbf{x}))$, are special cases. of this.

Prop. 2. [type 2]

Ψ is a well-representative form of K if $\Gamma \vdash \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))]) \vee \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))])$ and K and Ψ are tuples of n sub-tuples of predicates, $K = K_1, \cdots, K_n$ and $\Psi = \Psi_1, \cdots, \Psi_n$, where Kk, Ψ_k are tuples of the same number of predicates and *no predicate symbols in* $K_k, \cdots, K_n$ *occur in* Ψ_k.

Proof. Assume that $\Gamma \vdash \forall \mathbf{x}.(\Psi(\mathbf{x}) \equiv (\Psi(\mathbf{x}))[\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))])$. Then $[\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))] \cdot [\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))] = [\Psi]$. Any predicate in K does not occur in $\Psi[\Psi]^n$, so $\Psi[\Psi]^{n+1} = \Psi[\Psi]^n$. Therefore, $\Gamma \vdash \forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))([\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))] \cdot [\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))])^{n+1}$.

Later, in example 5, we will find an example of type 2.

Theorem 2. If Ψ is a well-representative form of K, then

1) any instance of ascription of K to Ψ preserves consistency and

2) any instance of ascription is true in all the most Ψ-tending models in K.

Proof. From the assumption of this theorem we can assume the existence of a free substitution Θ_K such that $\Gamma \vdash \forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))\Theta_K$. Let Ψ satisfy the ascribable condition, that is, H1) $\Gamma \vdash \Gamma[\lambda \mathbf{x}.(K(\mathbf{x}) \triangle \Psi(\mathbf{x}))] \wedge \Gamma[\lambda \mathbf{x}.(K(\mathbf{x}) \triangledown \Psi(\mathbf{x}))]$, then, using H1) and Kleene's theorem repeatedly $\Gamma \vdash \Gamma\Theta_K$. So by theorem 1 $\Gamma \cup \{\forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))\}$ is consistent, which proves 1). Now by completeness of the deduction system we can guarantee the existence of some model M of Γ such that $M \models \forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))$. It is clear that M is the very most Ψ-tending model in K from its definition, and any model N of Γ such that $N \models \neg\forall \mathbf{x}.(K(\mathbf{x}) \equiv \Psi(\mathbf{x}))$ is not more Ψ-tending than M. So 2) is proved.

By Theorems 1, when the well-representative condition is satisfied, if the antecedent of ascription schema is satisfied, then we are assured of the existence of most Ψ-tending models of K. Note that the result of theorem 1 can be applied to circumscription, and we can similarly think of the well-representaive form in circumscription. Lifschitz showed that circumscription preserves consistency when Γ is a set of *almost universal* formulas [4], which is a generalized class of *separable* formulas he proposed himself [3] and *universal* formulas proposed by Etherington[1]. Note that this condition governs Γ, while the well-representative condition governs the predicates which ascription relativizes. But the couples of predicates intended in [3] to be relativized by circumscription under the separability condition satisfy its well-representative condition. Because the separability condition require any predicate in Ψ not to contain any predicate in K, which satisfies the condition of type 1. From this standpoint, the well-representative condition is a weaker condition than the separability condition. When the well-representative condition is satisfied, if the antecedent of circumscription schema is satisfied, even with no minimal model, a most Ψ-tending model exists and circumscription preserves consistency.

5. What reasoning can ascription formalize?

In this section we will describe the types of reasoning that can be formalized by ascription. As mentioned at the beginning of this paper, ascription represents the flexible notion that the interpretation of a certain property K will lie between the extremes of the two. First we give these extremes. They will be useful in understanding the flexibility of the properties of ascription. Then we describe two types of reasoning, analogy and induction. Ascription is a form of some kinds of analogy and induction. Ascription seems to be especially characterized in formalizing these types of reasoning based on generalization from knowledge on individual instances. These are important because they are closely related to human learning abilities. Finally, we show that ascription is also a form of common sense reasoning.

5.1 Reasoning in the extremes, circumscription and inscription

Our notion of ascription involves that of *predicate (parallel)* circumscription proposed by John McCarthy [6]. Indeed, under some conditions, any theorems of a theory with predicate circumscription are also theorems of our theory with some ascription. Now, we do not know of the essential condition for ascription to involve circumscription in the above sense. But we do have a sufficient condition.

We can derive two significant products from ascription. One product is predicate circumscription, which formalizes conjectural reasoning based on the closed-world assumption. Its model, called most K_{min}-tending model, corresponds to the minimal model. The other is called inscription in this paper (indeed, both circumscription and inscription are included in formula circumscription, but we feel it is unsuitable to use the same term for them because of their quite different natures), which formalizes conjectural reasoning which generalizes some concepts. But inscription seems to generalize too strongly.

Let $\Omega_{K_*} = \{ \Psi \mid \Gamma \vdash \forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})) \wedge As(\Gamma, K \sim \Psi) \}$ and K_{min} be the conjunction of all elements of Ω_{K_*} i.e.

$$K_{min} = \wedge^{all} \Psi \qquad (\Psi \in \Omega_{K_*})$$

Now for any couple of elements of Ω_{K_*}, Ψ_1, Ψ_2, if $\Gamma \vdash \Gamma[\Psi_1][\Psi_2] \supset \Gamma[\lambda \mathbf{x}.(\Psi_1(\mathbf{x}) \wedge \Psi_2(\mathbf{x}))]$ then ascription involves circumscription. The reason is as follows. $\Gamma \vdash \forall \mathbf{x}.(\Psi_1(\mathbf{x}) \supset K(\mathbf{x})) \wedge \forall \mathbf{x}.(\Psi_2(\mathbf{x}) \supset K(\mathbf{x})) \wedge As(\Gamma, K \sim \Psi_1) \wedge As(\Gamma, K \sim \Psi_2)$, so $\Gamma \vdash \Gamma[\Psi_1] \wedge \Gamma[\Psi_2]$ (Notice that $\forall \mathbf{x}.(K(\mathbf{x}) \wedge \Psi_1(\mathbf{x}) \equiv \Psi_1(\mathbf{x}))$ $\forall \mathbf{x}.(K(\mathbf{x}) \vee \Psi_1(\mathbf{x}) \equiv K(\mathbf{x}))$, so $\Gamma \vdash \Gamma[\Psi_1]$. Similarly for Ψ_2). Assume that $\Gamma \vdash \Gamma[\Psi_1][\Psi_2] \supset \Gamma[\lambda \mathbf{x}.(\Psi_1(\mathbf{x}) \wedge \Psi_2(\mathbf{x}))]$, then $\Gamma \vdash \Gamma[\lambda \mathbf{x}.(\Psi_1(\mathbf{x}) \wedge \Psi_2(\mathbf{x}))]$. Therefore, $\Gamma \vdash As(\Gamma, K \sim \lambda \mathbf{x}.(\Psi_1(\mathbf{x}) \wedge \Psi_2(\mathbf{x})))$, that is, if $\Psi_1, \Psi_2 \in \Omega_{K_*}$ then $\lambda \mathbf{x}.(\Psi_1(\mathbf{x}) \wedge \Psi_2(\mathbf{x})) \in \Omega_{K_*}$. Finally, $K_{min} \in \Omega_{K_*}$ is proved. Then we ascribe K to K_{min} and obtain $\forall \mathbf{x}.(K(\mathbf{x}) \equiv K_{min}(\mathbf{x}))$. Thus, from $\Gamma\{K \sim K_{min}\}$

$$\forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})) \wedge \Gamma[\Psi] \supset \forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})). \qquad \cdots\cdots (2)$$

This is the same schema that McCarthy proposed as predicate circumscription.

And similarly, let $\Omega_{K^*} = \{ \Psi \mid \Gamma \vdash \forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})) \wedge As(\Gamma, K \sim \Psi) \}$ and K_{max} be the disjunction of all elements of Ω_{K^*}. For any couple of elements of Ω_{K^*}, Ψ_1, Ψ_2, assume that $\Gamma \vdash \Gamma[\Psi_1][\Psi_2] \supset \Gamma[\lambda \mathbf{x}.(\Psi_1(\mathbf{x}) \vee \Psi_2(\mathbf{x}))]$, then

$$\forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})) \wedge \Gamma[\Psi] \supset \forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})). \qquad \cdots\cdots (3)$$

(3) is called the *inscription schema*. Notice we can use either (2) or (3) on some K by replacing Ψ with any predicate which satisfies the above conditions. We can think of the predicate form which satisfies these conditions. For example, if Ψ is the predicate which can be transformed into on expression of the form, $\lambda\mathbf{x}.(K(\mathbf{x})\wedge G(\mathbf{x})\vee H(\mathbf{x}))$, where G, H are tuples of predicates in which no predicate symbols in K occur, then $\Gamma \vdash \forall\mathbf{x}.(\Psi_1(\mathbf{x}) \supset K(\mathbf{x})) \wedge \forall\mathbf{x}.(\Psi_2(\mathbf{x}) \supset K(\mathbf{x})) \wedge \Gamma[\Psi_1][\Psi_2] \supset \Gamma[\lambda\mathbf{x}.(\Psi_1(\mathbf{x})\wedge\Psi_2(\mathbf{x}))]$ and $\Gamma \vdash \forall\mathbf{x}.(K(\mathbf{x}) \supset \Psi_1(\mathbf{x})) \wedge \forall\mathbf{x}.(K(\mathbf{x}) \supset \Psi_2(\mathbf{x})) \wedge \Gamma[\Psi_1][\Psi_2] \supset \Gamma[\lambda\mathbf{x}.(\Psi_1(\mathbf{x})\vee\Psi_2(\mathbf{x}))]$. This predicate form is important because it is also a well-representative form of K as mentioned in Prop. 1.1.

Example 2. Various examples of circumscription are given in [6]. One interesting example of inscription can be found in the field of machine learning. Michalski proposed *selective generalization rules* [5], which consist of ten rules; the *dropping condition* rule, the *adding alternatives* rules, etc. If we can properly change these rules into the closed formulas of first order logic, they will be theorems of theory with inscription schema. This is partially explained bellow.

Most selective generalization rules are essentially described as follows.

$$\text{From } \forall\mathbf{x}.(\ \Phi(\mathbf{x}) \supset K(\mathbf{x})\ ) \wedge \forall\mathbf{x}.(\ \Phi(\mathbf{x}) \supset \Psi(\mathbf{x})\ ),$$
$$\text{infer } \forall\mathbf{x}.(\ \Psi(\mathbf{x}) \supset K(\mathbf{x})\ ).$$

Now let Γ be $\{\ \forall\mathbf{x}.(\ \Phi(\mathbf{x}) \supset K(\mathbf{x})\ ),\ \forall\mathbf{x}.(\ \Phi(\mathbf{x}) \supset \Psi(\mathbf{x})\ )\ \}$, then $\Gamma[K] \vdash \forall\mathbf{x}.(\ K(\mathbf{x}) \supset K(\mathbf{x})\vee\Psi(\mathbf{x})\ )\wedge\Gamma[\lambda\mathbf{t}.(K(\mathbf{t})\vee\Psi(\mathbf{t}))]$. Therefore,

$$\{\ \forall\mathbf{x}.(\ \Phi(\mathbf{x}) \supset K(\mathbf{x})\ ),\ \forall\mathbf{x}.(\ \Phi(\mathbf{x}) \supset \Psi(\mathbf{x})\ )\ \}$$
$$\mid\!\sim \forall\mathbf{x}.(\ \Psi(\mathbf{x}) \supset K(\mathbf{x})\ ).$$

5.2. Analogical inference

Ascription is a form of a certain class of analogy. According to the notion of ascription, analogy is considered as follows. When **a** resembles **b**, where **a** and **b** are tuples of entities, we consider **a** and **b** to have some common property Ψ. And now let **a** have some property K relevant to Ψ in that K and Ψ satisfy the ascribable condition. Then we can infer that **b** also has the property K. Here, to satisfy the ascribable condition means at least that we do not know the fact that **b** does not have the property K.

In most cases of formalization of analogy, the treatment of resemblance is unsatisfying. Resemblance is regarded as an atomic relation which cannot be explained. We may say "**a** is like **b**" and "**b** is like **c**", but likeness may be used in different senses. The inference "**a** is like **c**", derived by a rule like modus ponens, seems to be against our intuition in general. The problem seems to be that they ignore the common properties on the basis of which we consider that "**a** is like **b**". Moreover, in the case of analogizing K(**b**) from the fact { $\Psi(\mathbf{a})$, $\Psi(\mathbf{b})$, $K(\mathbf{a})$ }, the relation between K and Ψ should not be ignored, and K and Ψ must satisfy some condition. Let us take a example. A man is like a firework in that both have short lives. Yet we can never infer that a firework can love someone like a man. If the condition, that whatever we know to be capable of loving someone has a short life, is satisfied, then the inference that a firework can love someone like a man may be justified. And if a further condition, that whatever we know to be incapable of loving someone has a long life or is immortal, is satisfied, then it may be even more secure. The ascribable condition requires that these two conditions must be satisfied.

Example 3. Let Γ be "Hector is animate and would be sad if he were burnt, and if Brutus were burnt, he would be sad,too." Namely,

$$\Gamma = \{\text{Burnt(hector)} \supset \text{Sad(hector)},\ \ \text{Animate(hector)},\ \ \text{Burnt(brutus)} \supset \text{Sad(brutus)}\ \}.$$

Clearly $\Gamma \vdash \text{As}(\Gamma, \text{Animate} \sim \lambda x.(\text{Burnt}(x) \supset \text{Sad}(x)))$, therefore

$$\forall x.((\text{Burnt}(x) \supset \text{Sad}(x)) \equiv \text{Animate}(x)).$$

This says that whoever is sad when burnt is animate. So

$$\Gamma \mid\!\sim \text{Animate(brutus)}.$$

Namely, this reasoning, "If Hector and Brutus are burnt then both are sad, and to this extent Hector and Brutus are like each other. Now, Hector is animate so Brutus may also be so", is then a kind of analogy.

5.3. Inductive inference

Readers may have already noticed that in a theory with the ascription schema it is possible to reason inductively.

Example 4. Let Γ consist of some instances.

$$\Gamma \equiv \{\ \text{Ruddy-faced(matsumoto-san,oneday)},\ \ \text{Ruddy-faced(matsumoto-san,today)},\ \ \text{Cold(oneday)},\ \text{Cold(today)}\ \}$$

Then $\Gamma \vdash \text{As}(\Gamma, \text{Cold} \sim \lambda x.\text{Ruddy-faced}(\text{matsumoto-san}, x))$, therefore

$\Gamma \mid\sim \forall x.(\mathrm{Cold}(x) \equiv \text{Ruddy-faced}(\text{matsumoto-san},x))$.

This means that if the system knows it is cold, then it guesses Matsumoto-san will be ruddy-faced, and if he is ruddy-faced, then it expects a cold day. Moreover, in this example, we add the new predicate 'all' which expresses the property of the whole domain, as McCarthy proposed [6], and let the new extended theory be Γ''. Namely $\Gamma'' \equiv \Gamma \cup \{\forall x.\mathrm{all}(x)\}$. And then $\Gamma'' \vdash \mathrm{As}(\Gamma,\mathrm{all}\sim\lambda x.\text{Ruddy-faced}(\text{matsumoto-san},x))$, so

$$\Gamma'' \mid\sim \forall x.(\mathrm{all}(x) \equiv \text{Ruddy-faced}(\text{matsumoto-san},x)),$$

and therefore

$$\Gamma'' \mid\sim \forall x.(\text{Ruddy-faced}(\text{matsumoto-san},x)).$$

This means that if the system does not know of a day when Matsumoto-san was not ruddy faced, then it may guess that he is always ruddy-faced.

5.4 Common Sense Reasoning

Ascription can also be a form of common sense reasoning as well as formula circumscription.

Example 5. McCarthy proposed a predicate 'ab' [7], meaning abnormality, to handle common sense reasoning. Here 'Ab_n' is used in a similar sense. Let Γ be as follows. We want to know whether a bird P-suke can fly or not.

$$\begin{aligned}\Gamma \equiv \{\ & \forall x.(\neg \mathrm{Ab}_1(x) \supset \neg \mathrm{Fly}(x)),\\ & \forall x.(\mathrm{Plane}(x) \supset \mathrm{Ab}_1(x)),\\ & \forall x.(\mathrm{Bird}(x) \supset \mathrm{Ab}_1(x)),\\ & \forall x.(\mathrm{Plane}(x) \wedge \neg \mathrm{Ab}_2(x) \supset \mathrm{Fly}(x)),\\ & \forall x.(\mathrm{Bird}(x) \wedge \neg \mathrm{Ab}_3(x) \supset \mathrm{Fly}(x)),\\ & \forall x.(\mathrm{Penguin}(x) \supset \mathrm{Ab}_3(x)),\\ & \forall x.(\mathrm{Bird}(x) \wedge \mathrm{Dead}(x) \supset \mathrm{Ab}_3(x)),\\ & \forall x.(\mathrm{Penguin}(x) \wedge \neg \mathrm{Ab}_4(x) \supset \neg \mathrm{Fly}(x)),\\ & \forall x.(\mathrm{Bird}(x) \wedge \mathrm{Dead}(x) \wedge \neg \mathrm{Ab}_5(x) \supset \neg \mathrm{Fly}(x)),\\ & \mathrm{Bird}(\text{p suke}) \quad \}\end{aligned}$$

First we try to decide what is abnormal and what can fly.

$$\begin{aligned}&\Gamma \vdash\\ &\Gamma[\lambda x.(\mathrm{Fly}(x) \wedge (\mathrm{Plane}(x) \vee \mathrm{Bird}(x) \wedge \neg \mathrm{Penguin}(x) \wedge \neg \mathrm{Dead}(x))),\\ &\quad \lambda x.(\mathrm{Ab}_1(x) \wedge (\mathrm{Plane}(x) \vee \mathrm{Bird}(x))),\end{aligned}$$

$\lambda x.(Ab_2(x) \vee (false)),$
$\lambda x.(Ab_3(x) \vee (Penguin(x) \vee Bird(x) \wedge Dead(x))),$
$\lambda x.(Ab_4(x) \wedge (Penguin(x) \wedge Plane(x))),$
$\lambda x.(Ab_5(x) \wedge (Bird(x) \wedge Dead(x) \wedge Plane(x)))].$

And similarly

$$
\begin{aligned}
&\Gamma \vdash \\
&\Gamma[\lambda x.(Fly(x) \vee (Plane(x) \vee Bird(x) \wedge \neg Penguin(x) \wedge \neg Dead(x))), \\
&\quad \lambda x.(Ab_1(x) \vee (Plane(x) \vee Bird(x))), \\
&\quad \lambda x.(Ab_2(x) \wedge (false)), \\
&\quad \lambda x.(Ab_3(x) \wedge (Penguin(x) \vee Bird(x) \wedge Dead(x))), \\
&\quad \lambda x.(Ab_4(x) \vee (Penguin(x) \wedge Plane(x))), \\
&\quad \lambda x.(Ab_5(x) \vee (Bird(x) \wedge Dead(x) \wedge Plane(x)))].
\end{aligned}
$$

Thus by ascription we can get the candidates of the properties, $Ab_1, \cdots, Ab_5$ and Fly. Now we assume that P-suke is as normal as possible, that is we try to minimize its abnormality. Minimizing abnormality corresponds to supplementing lack of knowledge with common sense knowledge. Finally, the instance of ascription is

$$
\begin{aligned}
&\Gamma[\lambda x.(Plane(x) \wedge (false)), \\
&\quad \lambda x.(Penguin(x) \wedge (false)), \\
&\quad \lambda x.(Dead(x) \wedge (false)), \\
&\quad \lambda x.(Fly(x) \wedge (Plane(x) \vee Bird(x) \wedge \neg Penguin(x) \wedge \neg Dead(x))), \\
&\quad \lambda x.(Ab_1(x) \wedge (Plane(x) \vee Bird(x))), \\
&\quad \lambda x.(Ab_2(x) \vee (false)), \\
&\quad \lambda x.(Ab_3(x) \vee (Penguin(x) \vee Bird(x) \wedge Dead(x))), \\
&\quad \lambda x.(Ab_4(x) \wedge (Penguin(x) \wedge Plane(x))), \\
&\quad \lambda x.(Ab_5(x) \wedge (Bird(x) \wedge Dead(x) \wedge Plane(x)))] \\
&\wedge \\
&\Gamma[\lambda x.(Plane(x) \vee (false)), \\
&\quad \lambda x.(Penguin(x) \vee (false)), \\
&\quad \lambda x.(Dead(x) \vee (false)), \\
&\quad \lambda x.(Fly(x) \vee (Plane(x) \vee Bird(x) \wedge \neg Penguin(x) \wedge \neg Dead(x))), \\
&\quad \lambda x.(Ab_1(x) \vee (Plane(x) \vee Bird(x))), \\
&\quad \lambda x.(Ab_2(x) \wedge (false)), \\
&\quad \lambda x.(Ab_3(x) \wedge (Penguin(x) \vee Bird(x) \wedge Dead(x))), \\
&\quad \lambda x.(Ab_4(x) \vee (Penguin(x) \wedge Plane(x))), \\
&\quad \lambda x.(Ab_5(x) \vee (Bird(x) \wedge Dead(x) \wedge Plane(x)))] \\
&\supset \\
&\quad \forall x. \neg Plane(x) \wedge \forall x. \neg Penguin(x) \wedge \forall x. \neg Dead(x) \\
&\wedge \forall x.(Fly(x) \equiv (Plane(x) \vee Bird(x) \wedge \neg Penguin(x) \wedge \neg Dead(x)) \\
&\wedge \cdots.
\end{aligned}
$$

So

$$\Gamma \mid\!\sim \forall x.(Fly(x) \equiv Bird(x)).$$

And

$$\Gamma \mid\sim \text{Fly(p-suke)}.$$

6.Conclusion and remarks

As described above, ascription uniformly formalizes diverse and flexible conjectural reasoning performed by humans. But, of course, there still remain more difficult problems on its use. How do we, humans, use these various types of reasoning properly? Our conclusions will often contradict each other depending on how we interpret our knowledge about a certain property K; in a narrow sense, as in circumscription, or in a broad sense, as in analogy. This problem is deeply relevant to human preference and lies beyond the scope of our logic. We have not considered this much, but it seems that when we have less instances of K, we prefer a narrow interpretation, and that when we have sufficient instances of K, we prefer a broad interpretation. Considered from the viewpoint of ascription, this seems to correspond more or less to the situation that there are, roughly speaking, so many various dubious candidates for Ψ to K in the former case. Indeed, it will be difficult to choose an adequate Ψ, but Kmin is one of the well-founded candidates. In the latter case, because we get more information on K, there are fewer candidates so it seems to be easier to choose. Anyway, an adequate Ψ will usually be given in a moderate sense, i.e., neither in the narrowest nor in the broadest sense. We believe that ascription is a general form which can cover any proper interpretation of K between one extreme and another.

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