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A Form of Conjectural Reasoning on Equivalence---Ascription: —Preliminary Roport—

by

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A Form of Conjectural Reasoning on Equivalence --- Ascription:

PRELIMINARY REPORT

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Abstract

A logical framework is proposed that draws hypotheses to deduce properties of some unknown facts by relativizing and generalizing already acquired knowledge. It is called *ascription*. Ascription formalizes such flexible conjectural reasoning. Interestingly, ascription draws some inferences by induction, analogy, circumscription, and so on.

1.Introduction

Computer systems with capabilities of deductive inference will release man from the troublesome tasks of a procedural programming and be able to solve problems which he gives them only declaratively. But deductive inference is deduction of properties of individuals from given general knowledge, so it cannot provide effective consequences about facts that are unexpected and not included in general knowledge. This means that deductive inference cannot make a significant contribution to solving our software crisis. We should remember unexpected facts always exist and deductive inference is helpless with respect to them. One promising approach is reasoning by relativizing and generalizing already acquired knowledge so that it can be applied to unexpected circumstances.

Work related to this kind of reasoning was done by John McCarthy et al. *Circumscription* proposed there [5,6] is a form of such conjectural reasoning as is done by humans and based on a closed-world assumption. This work is exciting and interesting, but it seems that it can explain only a small part of human flexible reasoning and that there still remain some very important aspects which we should not ignore. These are analogy, induction and other reasoning which strongly relativizes and generalizes knowledge. Such reasoning is closely related to human learning capabilities. We have studied reasoning from this point of view and propose a logical framework, called *ascription*, which is a form of such conjectural reasoning. Intuitively, ascription represents the flexible notion that interpretation of a certain property K lies between two extremes; one, similar to circumscription, that the only demonstrated positive instances of K are all instances satisfying K; and the other, that all except the demonstrated negative instances satisfy K. More precisely we will show this as follows.

Ascription is based on the following notion. *If all the entities that can be shown to have a property K by reasoning from already acquired knowledge Γ can also be shown to have a property Ψ, and all the entities that can be shown not to have the property K can also be* *shown not to have the property Ψ, the property K may be equivalent to Ψ.* Namely when all the positive instances of K that can be shown to be so are the positive instances of Ψ, and similarly all the negative instances of K are the negative instances of Ψ, we can assume the equivalence of K and Ψ.

2.Ascription schema

In this paper we write **t** instead of a tuple of finite terms for brevity. For example, a formula A(**x**) stands for A(x1,···,xj) and the quantifier ∀**x**. stands for ∀x1.···.xk. By a finite set of formulas { A1, A2, ··· , Am } we mean a formula A1∧A2∧···∧Am.

We regard A(**x**) as a formula of first order logic in which a tuple **x** of individual variables occur free, where A is a predicate standing for a predicate symbol or a λ-expression λ**t**.(F(**t**)) of a formula F(**t**) of first order logic. Let K be a tuple of distinct predicate symbols, K1,···,Kn and Ψ a tuple of predicates, Ψ1,···,Ψn, where Ki and Ψi have the same arity. By [Ψ/K], representing [Ψ1/K1,···,Ψn/Kn] and usually abbreviated [Ψ], we mean a substitution. We write A[Ψ/K](**x**) for the result of replacing simultaneously each occurrence Ki in A(**x**) by Ψi. And similarly, [λ**t**.(Ψ(**t**))] stands for [λ**t**.(Ψ1(**t**)),···,λ**t**.(Ψn(**t**))], and ∀**x**.(K(**x**) ≡ Ψ(**x**)) stands for ∀**x**.(K1(**x**) ≡ Ψ1(**x**))∧···∧∀**x**.(Kn(**x**) ≡ Ψn(**x**)) (where ∀**x**.(Ki(**x**) ≡ Ψi(**x**)) means ∀**x**.(Ki(**x**) ⊃ Ψi(**x**)) ∧ ∀**x**.(Ki(**x**) ⊂ Ψi(**x**))).

Definition [*Ascription schema*].

Let K and V be tuples of distinct predicate symbols disjoint with each other (V may be empty, but K must not). And let Γ be a set of closed formulas of first order logic containing all predicates in K and V. The *ascription of K to Ψ in Γ[K,V] with variable V* is the schema

$$\Gamma[\lambda \mathbf{t}.(K(\mathbf{t})\wedge\Psi(\mathbf{t})),Y] \;\wedge\; \Gamma[\lambda \mathbf{t}.(K(\mathbf{t})\vee\Psi(\mathbf{t})),Y] \;\supset\; \forall \mathbf{x}.(\, K(\mathbf{x}) \equiv \Psi(\mathbf{x})\,). \quad \cdots \quad (1)$$

Here Ψ and Y are tuples of predicates which have the same arity as the corresponding predicates in K and V. We call the formula on the left side of this schema the *ascribable condition* writing As(Γ,K~Ψ,Y/V).

Γ[λ**t**.(K(**t**)∧Ψ(**t**)),Y] express the assumption that all the tuples that can be shown to have a certain property K by reasoning from certain facts Γ can also be shown to have a certain property Ψ. Γ[λ**t**.(K(**t**)∨Ψ(**t**)),Y] is, as far as K is concerned, equivalent to the result of replacing ¬K by λ**t**.(¬K(**t**)∧¬Ψ(**t**)). Namely Γ[λ**t**.(K(**t**)∨Ψ(**t**)),Y] express the assumption that all the tuples that can be shown not to have a property K by reasoning, can also be shown not to have a certain property Ψ. When we can assume that both Γ[λ**t**.(K(**t**)∧Ψ(**t**)),Y] and Γ[λ**t**.(K(**t**)∨Ψ(**t**)),Y] are true, (1) lets us conclude the formula on the right side, namely that K is equivalent to Ψ.

Now, when the formula p follows from a set of formulas Γ by a natural deduction system, we write Γ ⊢ p. Let $\Gamma_{h-1}^*\{K^h \sim \Psi^h\}$ be $\Gamma_{h-1}^* \cup \{ As(\Gamma_{h-1}^*, K^h \sim \Psi^h, Y^h/V^h) \supset \forall \mathbf{x}.(K^h(\mathbf{x}) \equiv \Psi^h(\mathbf{x}))\}$ (h = 1,2,···) and Γ_0^* be Γ. Let Γ_h^* be $\Gamma_{h-1}^*\{K^h \sim \Psi^h\}$, written $\Gamma\{K^1 \sim \Psi^1, \cdots, K^h \sim \Psi^h\}$. If a finite number n exists such that $\Gamma_n^* \vdash p$, we will write $\Gamma \mid\!\sim_{K^1\sim\Psi^1,\dots,K^n\sim\Psi^n} p$ and usually abbreviate this as $\Gamma \mid\!\sim p$.

Example 1. Let Γ be some relations among 'life', 'mammal' and 'human' and a few instances about 'homoiothermal'. We want to know what 'homoiothermal' is. Ascription shows a candidate of the concepts equivalent to 'homoiothermal'. Γ may be

$$
\begin{array}{l}
\Gamma = \Gamma[\mathrm{Homoiothermal}] \\
\equiv \{\ \forall x.(\mathrm{Human}(x) \supset \mathrm{Mammal}(x))\ , \\
\quad \forall x.(\mathrm{Mammal}(x) \supset \mathrm{Life}(x))\ , \\
\quad \exists x.(\neg \mathrm{Human}(x) \wedge \mathrm{Mammal}(x))\ , \\
\quad \exists x.(\neg \mathrm{Mammal}(x) \wedge \mathrm{Life}(x))\ , \\
\quad \exists x.(\mathrm{Human}(x) \wedge \mathrm{Homoiothermal}(x))\ , \\
\quad \exists x.(\mathrm{Life}(x) \wedge \neg \mathrm{Homoiothermal}(x))\ \}.
\end{array}
$$

First we check whether 'mammal' can be equivalent to 'homoiothermal' or not, namely check the ascription condition As(Γ,Homoiothermal~Mammal).

$$
\begin{array}{l}
\Gamma[\lambda x.(\mathrm{Homoiothermal}(x) \wedge \mathrm{Mammal}(x))] \equiv \\
\{ \quad \forall x.(\mathrm{Human}(x) \supset \mathrm{Mammal}(x))\ , \\
\quad \forall x.(\mathrm{Mammal}(x) \supset \mathrm{Life}(x))\ , \\
\quad \exists x.(\neg \mathrm{Human}(x) \wedge \mathrm{Mammal}(x))\ , \\
\quad \exists x.(\neg \mathrm{Mammal}(x) \wedge \mathrm{Life}(x))\ , \\
\quad \exists x.(\mathrm{Human}(x) \\
\quad\ \wedge (\mathrm{Homoiothermal}(x) \wedge \mathrm{Mammal}(x)))\ , \\
\quad \exists x.(\mathrm{Life}(x) \\
\quad \wedge \neg(\mathrm{Homoiothermal}(x) \wedge \mathrm{Mammal}(x)))\ \}
\end{array}
$$

$$
\begin{array}{l}
\Gamma[\lambda x.(\mathrm{Homoiothermal}(x) \vee \mathrm{Mammal}(x))] \equiv \\
\{ \cdots, \exists x.(\mathrm{Human}(x) \\
\quad\ \wedge (\mathrm{Homoiothermal}(x) \vee \mathrm{Mammal}(x)))\ , \\
\quad \exists x.(\mathrm{Life}(x) \\
\quad \wedge \neg(\mathrm{Homoiothermal}(x) \vee \mathrm{Mammal}(x)))\ \}
\end{array}
$$

Clearly $\Gamma \vdash \Gamma[\lambda x.(\mathrm{Homoiothermal}(x) \wedge \mathrm{Mammal}(x))] \wedge \Gamma[\lambda x.(\mathrm{Homoiothermal}(x) \vee \mathrm{Mammal}(x))]$, so

$$
\begin{array}{l}
\Gamma \cup \{\ \mathrm{As}(\Gamma,\mathrm{Homoiothermal}\sim\mathrm{Mammal}) \\
\quad \supset \forall x.(\mathrm{Homoiothermal}(x) \equiv \mathrm{Mammal}(x))\ \} \\
\vdash \forall x.(\mathrm{Homoiothermal}(x) \equiv \mathrm{Mammal}(x)).
\end{array}
$$

This shows that 'homoiothermal' may be 'mammal'. And therefore we get

$$
\begin{array}{l}
\Gamma \mid\!\sim \quad \forall x.(\mathrm{Homoiothermal}(x) \equiv \mathrm{Mammal}(x)) \\
\qquad \wedge\ \forall x.(\mathrm{Human}(x) \supset \mathrm{Homoiothermal}(x))
\end{array}
$$

$\wedge \forall x.(\text{Homoiothermal}(x) \supset \text{Life}(x))$.

Notice that these inferences cannot be derived by circumscription [5] (even *formula* circumscription [6]).

Now if we add the axiom

$$\exists x.(\neg \text{Homoiothermal}(x) \wedge \text{Mammal}(x))$$

then $\Gamma[\lambda x.(\text{Homoiothermal}(x) \vee \text{Mammal}(x))]$ is inconsistent, so $\forall x.(\text{Homoiothermal}(x) \equiv \text{Mammal}(x))$ is not a theorem of the extended theory Γ' ,where $\Gamma' = \Gamma \cup \{ \exists x.(\neg \text{Homoiothermal}(x) \wedge \text{Mammal}(x)) \}$. This shows that reasoning by ascription is non-monotonic.

3.Model theory of ascription

Definition [*Ψ-tending model* in K with variable V].

Let $M(\Gamma)$ and $N(\Gamma)$ be models of the sentence Γ. We say M is a more *Ψ-tending model* than N in K with variable V, writing $M \geqq_{K\sim\Psi,V} N$, if M and N have the same domain, and if all other predicate symbols not in K,V have the same extensions in M and N, but the extension of $\lambda x.(K(x) \wedge \Psi(x))$ in M includes its extension in N and the extension of $\lambda x.(\neg K(x) \wedge \neg \Psi(x))$ in M includes its extension in N.

Definition [*most Ψ-tending model* in K with variable V].

A model M of Γ is called *most Ψ-tending* in K with variable V iff $M' \geqq_{K\sim\Psi,V} M$ only if $M' =_{K\sim\Psi,V} M$ (where by " $M' =_{K\sim\Psi,V} M$ " we mean " $M' \geqq_{K\sim\Psi,V} M$ and $M' \leqq_{K\sim\Psi,V} M$ ").

4.On satisfiability of ascription

In this section we propose two consistency conditions which are sufficient conditions for ascription to preserve consistency, and introduce a class of most Ψ-tending model which preserves that any instance of ascription is true. When these conditions are satisfied, a consistent Γ cannot contradict the result drawn by ascription. One of these is on using ascription only once, called *CCPA* (consistency condition on parallel ascription), and the other is on using it more than once with the same Γ, called *CCSA* (consistency condition on sequential ascription).

Definition [*CCPA*]. By *CCPA* we mean the condition that Ψ *can be transformed into one of the expressions of the following form:* F, $\lambda t.(K(t) \wedge F(t))$ or $\lambda t.(K(t) \vee F(t))$, *where* F *is a tuple of predicates in which no predicate symbols in* K,V *occur.*

Theorem 1 (satisfiability of parallel ascription). Let $\Gamma[K,V]$ be consistent. When CCPA is satisfied, the following sentence is true;

if $\Gamma[K,V] \vdash \Gamma[\lambda t.(K(t) \wedge \Psi(t)),Y] \wedge \Gamma[\lambda t.(K(t) \vee \Psi(t)),Y]$ then $\Gamma[K,V] \cup \{\forall t.(K(t) \equiv \Psi(t))\}$ is consistent.

Theorem 2. When CCPA is satisfied, most Ψ-tending model is called *proper*. Any instance of the ascription in K to Ψ of Γ with variable V is true in all the proper most Ψ-tending model in K of Γ with variable V.

Definition [*CCSA*]. For some sequence of Ki,Vi ($i=1,\cdots,n,\ n\geqq 2$), by *CCSA* we mean the condition that *for all* i,j ($i,j=1,\cdots,n\ ,\ i\neq j$) Ki,Vi *are disjoint to* Kj,Vj *and for all* k ($k=1,\cdots,n$) Ψk *can be transformed into one of the expressions of the following form:* Fk, $\lambda t.(Kk(t)\wedge Fk(t))$ *or* $\lambda t.(Kk(t)\vee Fk(t))$, *where* Fk *is a tuple of predicates in which no predicate symbols in* Kk,Vk,…,Kn,Vn *occur*.

Theorem 3 (satisfiability of sequential ascription). Let Γ be consistent. If $\Gamma[Ki,Vi] \vdash \Gamma[\lambda t.(Ki(t)\wedge\Psi i(t)),Yi] \wedge \Gamma[\lambda t.(Ki(t)\vee\Psi i(t)),Yi]$ ($i=1,\cdots,n,\ n\geqq 2$) and CCSA are satisfied, then $\Gamma \cup \{\forall t.(\ K1(t) \equiv \Psi 1(t)\ ),\cdots,\forall t.(\ Kn(t) \equiv \Psi n(t)\ )\}$ is consistent and $\Gamma \mid\!\sim \forall t.(\ K1(t) \equiv \Psi 1(t)\ ) \wedge \cdots \wedge \forall t.(\ Kn(t) \equiv \Psi n(t)\ )$.

By Theorems 1 and 3, when CCPA or CCSA are satisfied we are assured of the existence of a proper most Ψ-tending model of K. Note that the result of theorem 1 can be applied to circumscription. Lifschitz showed that circumscription preserves consistency when Γ is a set of *almost universal* formulas [3], which is a generalized class of *separable* formulas he proposed himself [2] and *universal* formulas proposed by Etherington[1]. This condition guarantees the existence of a minimal model. Note that this condition is on Γ, while CCPA (and CCSA) is on the predicates which ascription relativizes. But the couples of predicates which are intended in [2] to be relativized by circumscription under separability condition satisfy CCPA. From this standpoint, CCPA is a weaker condition than separability condition. When CCPA (and CCSA) is satisfied, even with no minimal model, a proper most Ψ-tending model exists and circumscription preserves consistency.

5.What reasoning can ascription formalize?

In this section we will describe what reasoning ascription can formalize. As mentioned above, ascription represents the flexible notion that the interpretation of a certain property K will lie between the extremes of the two. First we give these extremes. They will be useful in understanding the flexibility of the properties of ascription. Then we describe two types of reasoning, analogy and induction, which ascription is a form of. Ascription seems to be characterized by its formalizing these reasoning especially from knowledge on individual instances, namely described by formula with no variable terms. These reasoning is important because they are closely related to human learning ability. Last, it is a natural result, but we show that ascription is also a form of common sense reasoning.

5.1 Reasoning in the extremes, circumscription and inscription

The notion of our ascription involves that of *predicate* (*parallel*) circumscription proposed by John McCarthy [5]. Indeed, under CCPA, any theorems of a theory with the predicate circumscription are also theorems of our theory with the circumscription schema of ascription. We show this.

We can derive two significant products from ascription. One product is predicate circumscription, which formalizes such conjectural reasoning as is based on the closed-world assumption. Its model, called the most K_{min}-tending model, corresponds to the minimal model. The other is called inscription in this paper (indeed, both circumscription and inscription are included in formula circumscription, but we feel it is unsuitable to use the same term for them because of their quite different nature), which formalizes such conjectural reasoning as generalizes some concepts. But inscription seems to generalize too strongly.

Let $\Omega_{K_*} = \{ \Psi \mid \Gamma \vdash \forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})) \wedge As(\Gamma,K\sim\Psi) \}$ and $\Omega_{K^*} = \{ \Psi \mid \Gamma \vdash \forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})) \wedge As(\Gamma,K\sim\Psi) \}$,where K,Ψ satisfy CCPA. Let K_{min} be conjunction of all elements of Ω_{K_*} and K_{max} be disjunction of all elements of Ω_{K^*}. i.e.

$$K_{min} = \bigwedge^{all} \Psi \in \Omega_{K_*}$$
$$K_{max} = \bigvee^{all} \Psi \in \Omega_{K^*}$$

We can prove $K_{min} \in \Omega_{K_*}$ and $K_{max} \in \Omega_{K^*}$. Then we ascribe K to K_{min} or K_{max} , we obtain respectively $\forall \mathbf{x}.(K(\mathbf{x}) \equiv K_{min}(\mathbf{x}))$ or $\forall \mathbf{x}.(K(\mathbf{x}) \equiv K_{max}(\mathbf{x}))$. Thus the two schemata follow from $\Gamma\{K\sim K_{min}\}$ and $\Gamma\{K\sim K_{max}\}$ respectively, namely,

$$\forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})) \wedge As(\Gamma,K\sim\Psi) \supset \forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})) \qquad \cdots\cdots \quad (2)$$
$$\forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})) \wedge As(\Gamma,K\sim\Psi) \supset \forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})). \qquad \cdots\cdots \quad (3)$$

Now we assume that $\forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x}))$. Then $\forall t.(K(t)\wedge\Psi(t) \equiv \Psi(t))$. Therefore, $\Gamma[\lambda t.(K(t)\wedge\Psi(t))] \equiv \Gamma[\Psi]$. And since $\forall t.(K(t)\vee\Psi(t) \equiv K(t))$, $\Gamma[\lambda t.(K(t)\vee\Psi(t))] = \Gamma[K]$, which is precisely the set of axioms Γ and is true. Therefore, from the assumption and (2),

$$\forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})) \wedge \Gamma[\Psi] \supset \forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})). \qquad \cdots\cdots \quad (4)$$

This is the same schema that McCarthy proposed as predicate circumscription.

And similarly,

$$\forall \mathbf{x}.(K(\mathbf{x}) \supset \Psi(\mathbf{x})) \wedge \Gamma[\Psi] \supset \forall \mathbf{x}.(\Psi(\mathbf{x}) \supset K(\mathbf{x})). \qquad \cdots\cdots \quad (5)$$

We call (5) the *inscription schema*. Notice we can use either (4) or (5) on some K with replacing Ψ by any predicate (if CCPA is satisfied, they preserve consistency).

Example 2. We can see the various examples on circumscription in [5]. One interesting example on inscription can be seen in the field of machine learning. Michalski proposed *selective generalization rules* [4], which consists of ten rules; the *dropping condition* rule, the *adding alternatives* rules, etc. If we can properly change these rules into the closed formulas of first order logic, these rules will be the theorems of theory with inscription schema. Here we explain it partly.

Most of selective generalization rules are essentially described as follows.

$$\text{From } \forall \mathbf{x}.(\ \Phi(\mathbf{x}) \supset K(\mathbf{x})\ ) \wedge \forall \mathbf{x}.(\ \Phi(\mathbf{x}) \supset \Psi(\mathbf{x})\ ),$$
$$\text{infer } \forall \mathbf{x}.(\ \Psi(\mathbf{x}) \supset K(\mathbf{x})\ ).$$

Now let Γ be $\{\ \forall \mathbf{x}.(\ \Phi(\mathbf{x}) \supset K(\mathbf{x})\ ),\ \forall \mathbf{x}.(\ \Phi(\mathbf{x}) \supset \Psi(\mathbf{x})\ )\ \}$, then $\Gamma[K] \vdash \forall \mathbf{x}.(\ K(\mathbf{x}) \supset K(\mathbf{x}) \vee \Psi(\mathbf{x})\ ) \wedge \Gamma[\lambda \mathbf{t}.(K(\mathbf{t}) \vee \Psi(\mathbf{t}))]$. Therefore,

$$\{\ \forall \mathbf{x}.(\ \Phi(\mathbf{x}) \supset K(\mathbf{x})\ ),\ \forall \mathbf{x}.(\ \Phi(\mathbf{x}) \supset \Psi(\mathbf{x})\ )\ \}$$
$$|\sim \forall \mathbf{x}.(\ \Psi(\mathbf{x}) \supset K(\mathbf{x})\ ).$$

5.2. Analogical inference

Ascription is a form of a certain class of analogy. According to the notion of ascription, analogy is considered as follows. When **a** resembles **b**, where **a** and **b** are tuples of entities, we consider **a** and **b** to have some common property Ψ. And now let **a** have some property K relevant to Ψ in that K and Ψ satisfy the ascribable condition. Then we can infer that **b** has also the property K. Here, to satisfy the ascribable condition means at least that we do not know the fact that **b** does not have the property K.

We do not know of research on formalizing analogical inference which discussed the relation between resemblance between two instances and the property we want to ascribe (or deny) to the one of the instances by analogy. But this relation should not be ignored. Two properties like K and Ψ must satisfy some conditions. For example, a man is like a firework in that both have short lives. Yet we can never infer that a firework can love someone like a man. If the condition, that whatever we know to be capable of loving someone has a short life, is satisfied, then the inference that a firework can love someone like a man may be justified. And if a further condition, that whatever we know to be incapable of loving someone has a long life or is immortal, is satisfied, then it may be even more secure. The ascribable condition require that these two conditions are satisfied.

Example 3. Let Γ be "Hector is an organism (predicate, Life) and would be sad if he were burnt, and if Brutus were burnt, he would be sad,too." Namely,

$$\Gamma = \{\mathrm{Burnt(hector)} \supset \mathrm{Sad(hector)},\ \mathrm{Life(hector)},\ \mathrm{Burnt(brutus)} \supset \mathrm{Sad(brutus)}\ \}.$$

Clearly $\Gamma \vdash \mathrm{As}(\Gamma, \mathrm{Life} \sim \lambda x.(\mathrm{Burnt}(x) \supset \mathrm{Sad}(x)))$, therefore

$$\forall x.((\mathrm{Burnt}(x) \supset \mathrm{Sad}(x)) \equiv \mathrm{Life}(x)).$$

This says that whoever is sad when burnt is an organism. So

$$\Gamma\ |\sim\ \mathrm{Life(brutus)}.$$

Namely, this reasoning, "If Hector and Brutus are burnt then both are sad and in this point Hector and Brutus are like each other, now Hector is an organism so Brutus may also be so", shows itself as a kind of analogy.

5.3. Inductive inference

Readers may have already noticed that in a theory with the ascription schema it is possible to reason inductively.

Example 4. Let Γ consist of some instances.

$$\Gamma = \{ \text{Ruddy-faced(matsumoto-san,oneday)}, \text{Ruddy-faced(matsumoto-san,today)}, \text{Cold(oneday)}, \text{Cold(today)} \}$$

Then $\Gamma \vdash \text{As}(\Gamma, \text{Cold} \sim \lambda x.\text{Ruddy-faced(matsumoto-san},x))$, therefore

$$\Gamma \mid\!\sim \forall x.(\ \text{Cold}(x) \equiv \text{Ruddy-faced(matsumoto-san},x)\ ).$$

This means that if the system knows it is cold, then it guesses Matsumoto-san will be ruddy-faced, and if he is ruddy-faced, then it expects a cold day. Moreover, in this example, we add the new predicate 'all' which express the property of the whole domain, as McCarthy proposed [5], and let the new extended theory be Γ'. Namely $\Gamma' = \Gamma \cup \{\forall x.\text{all}(x)\}$. And then $\Gamma' \vdash \text{As}(\Gamma, \text{all} \sim \lambda x.\text{Ruddy-faced(matsumoto-san},x))$, so

$$\Gamma' \mid\!\sim \forall x.(\ \text{all}(x) \equiv \text{Ruddy-faced(matsumoto-san},x)\ ),$$

and therefore

$$\Gamma' \mid\!\sim \forall x.(\ \text{Ruddy-faced(matsumoto-san},x)\ ).$$

This means that if the system does not know of a day when he was not ruddy-faced, then it may guess that he is always ruddy-faced.

5.4 Common Sense Reasoning

Ascription with variables can be a form of common sense reasoning as well as circumscription.

Example 5. McCarthy proposed a predicate 'ab' [6], meaning abnormality, to handle common sense reasoning. Here 'Ab_n' is used in a similar sense. Let Γ be as follows. We want to know whether a bird p-suke can fly or not.

$$\Gamma = \{ \forall x.(\text{Bird}(x) \wedge \neg \text{Ab}_1(x) \supset \text{Fly}(x)), \forall x.(\text{Penguin}(x) \supset \text{Ab}_1(x)), \forall x.(\text{Penguin}(x) \wedge \neg \text{Ab}_2(x) \supset \neg \text{Fly}(x)), \text{Bird(p-suke)} \}$$

Now we assume that P-suke is as normal as possible. Then we decide what is abnormal and assume its minimality.

$$
\begin{aligned}
&As(\Gamma[Ab_1,Fly],\ Ab_1{\sim}Penguin,\\
&\qquad \lambda x.(Bird(x)\wedge\neg Penguin(x)\vee Fly(x))/Fly)\\
&\equiv\{\ \forall x.(Bird(x)\ \wedge\ \neg(Ab_1(x)\wedge Penguin(x))\\
&\qquad\quad \supset (Bird(x)\wedge\neg Penguin(x)\vee Fly(x)))\ ,\\
&\qquad \forall x.(Penguin(x)\ \ \supset (Ab_1(x)\wedge Penguin(x)))\ ,\\
&\qquad \forall x.(Penguin(x)\ \wedge\ \neg Ab_2(x)\\
&\qquad\quad \supset \neg(Bird(x)\wedge\neg Penguin(x)\vee Fly(x)))\ ,\\
&\qquad Bird(p\text{-}suke)\ \}\\
&\cup\{\ \forall x.(Bird(x)\ \wedge\ \neg(Ab_1(x)\vee Penguin(x))\\
&\qquad\quad \supset (Bird(x)\wedge\neg Penguin(x)\vee Fly(x)))\ ,\\
&\qquad \forall x.(Penguin(x)\ \ \supset (Ab_1(x)\vee Penguin(x)))\ ,\\
&\qquad \cdots\ \}
\end{aligned}
$$

It is easy to see that $\Gamma \vdash As(\ \Gamma[Ab_1,Fly]\ ,\ Ab_1{\sim}Penguin\ ,\ \lambda x.(Bird(x)\wedge\neg Penguin(x))/Fly\ )$. And similarly, $\Gamma \vdash As(\ \Gamma[Penguin]\ ,\ Penguin{\sim}false\ )$. Therefore, by theorem 3,

$$
\begin{aligned}
&\Gamma\{Ab_1{\sim}Penguin,Penguin{\sim}false\}\\
&\vdash\ \ \forall x.(Ab_1(x)\equiv Penguin(x))\wedge\forall x.\neg Penguin(x)\\
&\Gamma\ |\!\sim\ \ \forall x.\neg Ab_1(x)\\
&\Gamma\ |\!\sim\ \ Fly(p\text{-}suke).
\end{aligned}
$$

6.Conclusion and remarks

As described above, ascription uniformly formalizes diverse and flexible conjectural reasoning performed by humans. But ,of course, there still remain more difficult problems on its use. How do we, humans, use these various types of reasoning properly? Our conclusions will often contradict each other depending on how we interpret our knowledge about a certain property K; in a narrow sense, as in circumscription, or in a broad sense, as in analogy. This problem is deeply relevant to human preference and lies beyond the scope of our logic. We have not considered this much, but it seems that when we have less instances on K, we prefer a narrow interpretation, and that when we have sufficient instances on K, we prefer a broad interpretation. Considered from the viewpoint of ascription, this seems to correspond more or less to the situation that there are, roughly speaking, so many various dubious candidates for Ψ to K in the former case and indeed it will be difficult to choose an adequate Ψ, but Kmin is one of the well-grounded candidates. In the latter case, because we get more information on K, there are fewer candidates so it seems to be easier to choose. Anyway, an adequate Ψ will usually be given in a moderate sense, i.e. neither in the narrowest nor in the broadest sense. We believe that ascription is a general form which can cover any proper interpretation of K between one extreme and another.

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